The Life of the Collatz Conjecture

"The simplest impossible problem" -Jeffrey Lagarius

"Mathematics is not ready for such problems" -Paul Erdos

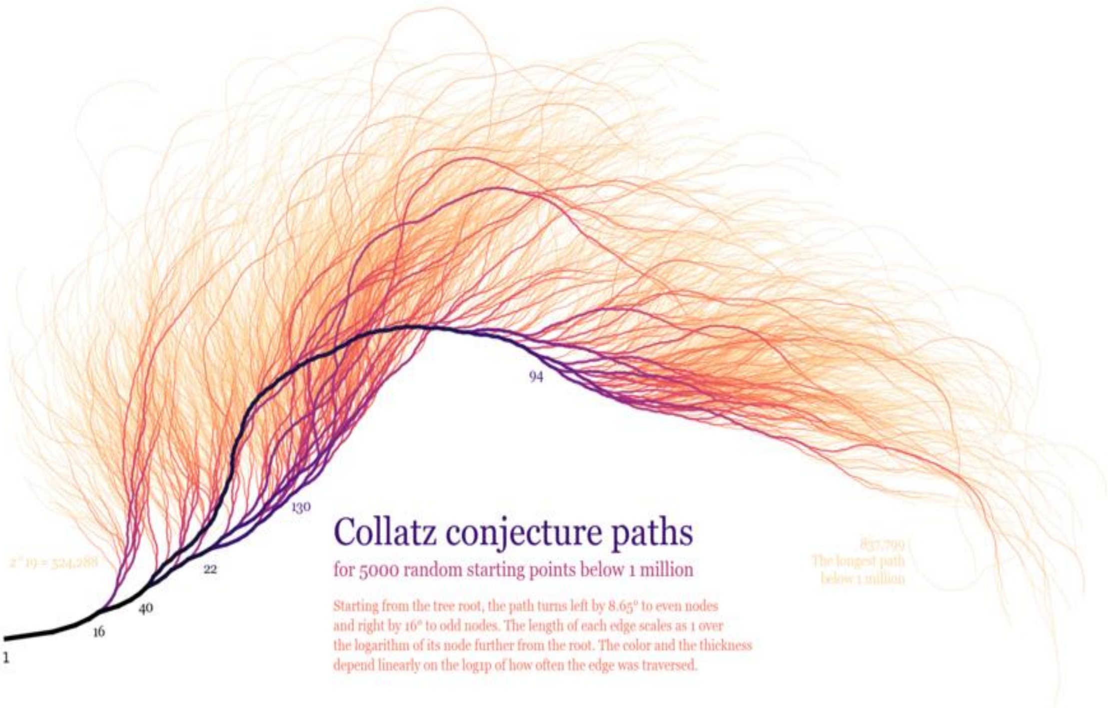

Figure 1: *Collatz conjecture visualisation* (Wikipedia, n.d.)

Introduction

Over time, mathematical problems almost feel alive. **John Conway** once commented, *"The Collatz problem is like a weed: it grows on its own and is impossible to kill."* Conway uses a metaphor that clearly describes the conjecture's strange behaviour: sequences grow abnormally yet always collapse to 1. He states its resistance to proof by being '*impossible to kill*', almost as if it is a living immortal creature.

But Conway's comparison raises an even bigger question:

<u>*What does it mean for a conjecture to be living?*</u>

To describe a conjecture as living is to understand that its meaning develops over time, not by changing the rule itself, but rather through the questions it creates. The Collatz conjecture imitates this lifecycle by its path of birth, growth, and inevitable death.

I: Birth

Every living thing has a birth - so does the Collatz conjecture. In 1937, Lothar Collatz, a German mathematician, proposed what would become one of the most unusually simple problems in mathematics. Its rules are straightforward:

Choose a positive integer.

If even, divide it by 2.

If odd, multiply it by 3 and add 1.

Repeat this with your new number.

For example, let's say we start with 22,

$$22 \to 11 \to 34 \to 17 \to 52 \to 26 \to 13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1 \ldots$$

Once the sequence reaches 4, it falls into a repeating cycle 4,2,1, occurring forever. The Collatz conjecture states that **every positive integer** incorporated into the equation will eventually reach 1(yet no proof exists).

Once proposed, mathematicians began experimenting different starting values. Earlier experimenting was carried out manually to see if any number escaped this pattern, but they quickly realised all numbers converged to 1. Next, they turned to modelling:

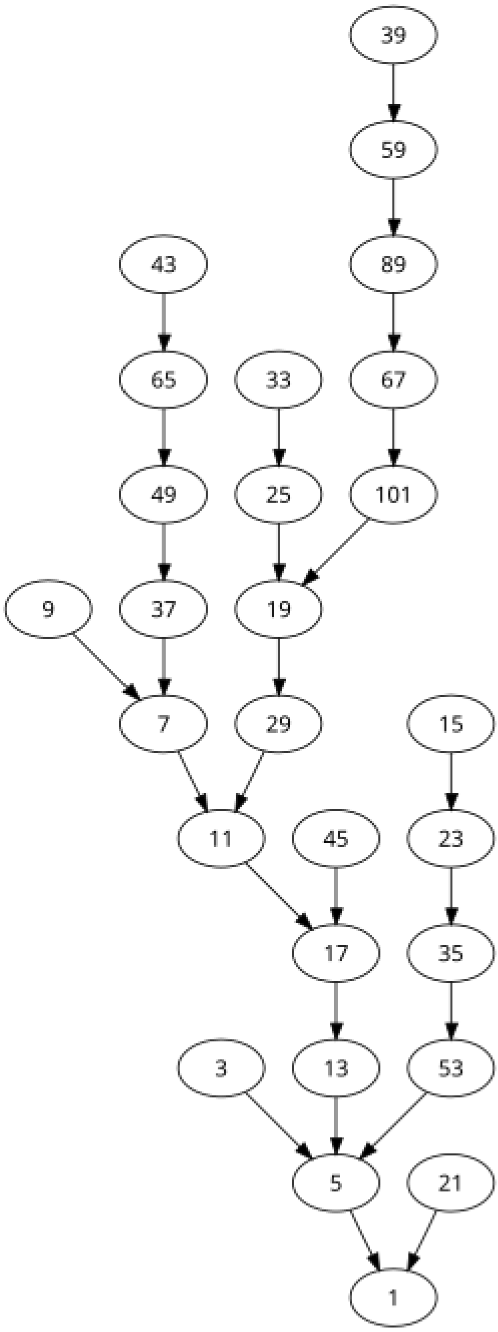

Figure 2: *Tree-like structure of Collatz sequences under the Syracuse map transformation*. (Wikipedia, n.d.)

The **Syracuse Map**, developed in the 1950s, combines the steps of the Collatz conjecture into one operation: $f(n) = \frac{3n+1}{2^k}$, (where 2^k is the highest power of 2 that divides 3n+1). When visualised, the structure resembles a tree rooted at 1, illustrating that for all small numbers tested, every possible route mysteriously will reach the root of the tree.

What is particularly extraordinary about the Collatz conjecture is the contrast between its simple formula and its extreme difficulty to solve. Many other famous mathematical problems, such as Fermat's Last Theorem or the Riemann Hypothesis require advanced mathematics to even understand! However, the Collatz rule can be easily explained using basic arithmetic, although even incorporating small numbers into a model we see how complex and lifelike it becomes.

This mystery is what births conjectures: the consistent strange patterns with no explanation causing us to spark new questions and question our own questions! In this way, the Collatz conjecture was not just created, it was born.

<u>II: Growth</u>

After its proposal the Collatz conjecture attracted the attention of many mathematicians. Its mysterious simplicity and unresolved nature made it hard to miss, and quickly became well recognised across the mathematical community. As researchers explored different starting values, they discovered that even small numbers could behave unexpectedly such as the number 27, which rises as high as 9232 before inevitably falling back to 1.

New models soon emerged to understand this behaviour:

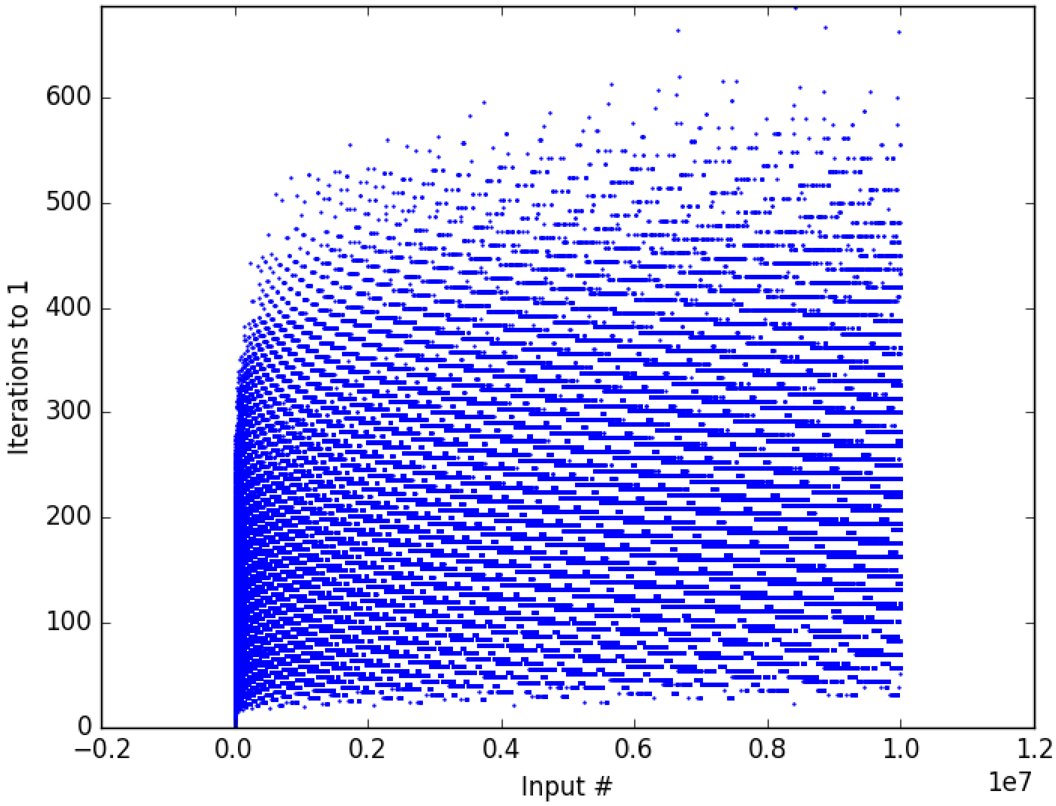

Figure 3: *Distribution of stopping times for Collatz sequences over large starting values*. (Wikipedia, n.d.)

This complexity becomes clearer when visualised. The diagram reveals an irregular and highly unpredictable pattern. Rather than forming a clear trend, the values fluctuate dramatically, with sudden spikes where certain numbers take longer to collapse. This suggests that the sequences don't follow a simple progression but behave randomly. But, despite this, every sequence returns to 1. This contrast between unpredictability and consistency supports the idea of the conjecture possessing a kind of life.

A second visualisation further highlights this behaviour:

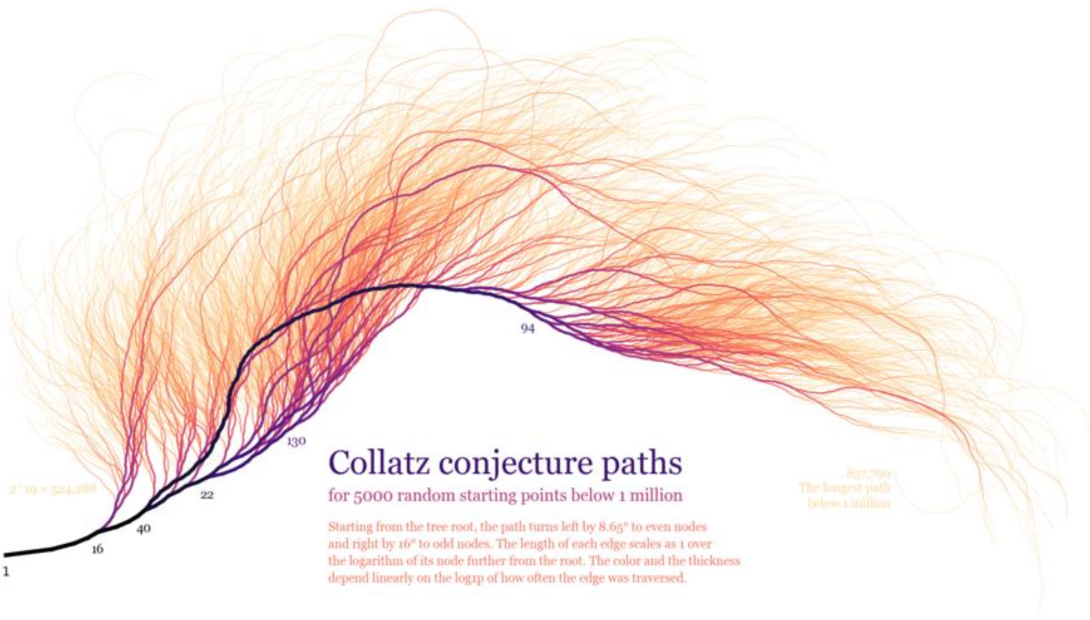

Figure 1: *Organic growth patterns of Collatz sequences using rotational mapping*. (Wikipedia, n.d.)

By representing each step as a small rotation – turning slightly in one direction for even numbers and another for odd numbers – the paths of sequences can be drawn as continuous lines. When applied to thousands of starting values, the result is a dense web of overlapping curves that appear almost organic in structure. The paths twist, branch, intersect intricately, resembling the growth of a living organism. Although every sequence follows the same rule, each produces a unique trajectory before inevitably converging to 1. This describes that the problem is not static, but evolves through each iteration, mirroring the behaviour of a living system.

Over time, this famous conjecture has intimidated many generations of mathematicians. Paul Erdős famously remarked, “Mathematics isn’t ready for such problems. “, reflecting the difficulty it presents. However, in 2019, Terence Tao decided to approach the Collatz conjecture from a new perspective. He focused on large-scale computational testing rather than a complete proof. He showed that almost all numbers follow the behaviour of this conjecture. To make this manageable, Tao had to ignore many different types of numbers. For example, he ignored multiples of 3, as these numbers typically are weeded out. He tested numbers up to 2^{68} (this is a higher number than grains of sand on the Earth!). Tao’s work demonstrated that, despite remaining unproven, sequences are not entirely random but follow underlying patterns.

Compared to other famous conjectures, this problem is highly unusual. Fermat’s last theorem required centuries of advanced mathematics before it was proven, while the Riemann hypothesis remains a deep problem connected to prime numbers. In contrast, the Collatz rule is simple yet shows extraordinary complexity. Its growth is therefore shown not just by the numbers it generates, but the mystery it creates.

Yet, despite decades of exploration, the conjecture surprisingly remains unsolved, raising a final question:

What does it mean for a mathematical problem to reach an end when it has neither been proven nor disproven?

<u>III: Death</u>

The most obvious way for a conjecture to 'die' is through proof or disproof. Once proven, it becomes a theorem. For example, Fermat's Last Theorem remained unsolved for over 350 years before finally being proven. Alternatively, a single counterexample can bring such idea to an abrupt end. In the case of the Collatz conjecture, this would mean discovering a number that never reaches 1, a sequence that grows infinitely, or a cycle different to 4, 2, 1.

Yet death in mathematics is not necessarily so definitive. With modern computation, the Collatz conjecture has been verified across extraordinary large ranges of numbers. This leads us to reflect more deeply: if a statement holds for every tested case, does there come a point where it is effectively proven, even without a formal proof? Unlike Fermat's last theorem, which reached a clear conclusion, the Collatz conjecture exists in a strange middle ground – being supported by overwhelming evidence but lacking a formal proof.

There is also a quieter form of death. Some ideas are never resolved, but forgotten. As mathematical focus shifts, some problems fade from attention, losing curiosity that once sustained them. In this sense, they do not end through resolution, but rather through neglection. Yet the Collatz conjecture resists this fate. Decades after creation, it continues to attract attention and challenge mathematicians, though this might not always be the case.

Perhaps this is what makes it truly alive. It hasn't been resolved like other conjectures, nor has it been abandoned, but instead, it perseveres. The Collatz has not reached its death (yet), and until it does, it will remain one of mathematics' most enduring living mysteries.

References:

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