# The limits of knowing: Why mathematics cannot prove everything

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This sentence is false.

That statement seems harmless enough, right? A bit of a strange opening for an essay perhaps, but it's just four ordinary words. Well, try deciding whether it's telling the truth, and you just might change your mind.

If it is true, then what it's saying must be correct - so it's false.

But if it's false, then it isn't false… which makes it true.

I'm sure you've realised the issue by now: whichever route you take, your reasoning turns back on itself, and you end up stuck in an endless loop. I suppose you could say it's the linguistic version of a dog chasing its own tail.

This logical trap is known as the Liar's Paradox, and despite over two millennia of debate, there is still no universally accepted solution. The paradox persists because the sentence talks about itself, and once you allow that, everything starts to unravel. By referring to its own truth value, it creates a contradictory cycle that classical logic cannot consistently resolve, and our reasoning starts tying itself up in knots. [1]

But since this is supposed to be an essay about mathematics, here's the real question: if natural language so easily succumbs to this idea of self-reference, can the same be said for mathematics? Many would assume not, and for good reason! Mathematics traditionally feels safer; numbers don't play word games and equations appear immune to paradox.

Kurt Gödel, on the other hand, decided that mathematics too can talk about itself. Up until this point, mathematics had seemed clear, precise, and removed from the messiness of language. David Hilbert, along with many others, believed that all of mathematics could be built on a complete and consistent set of axioms. [2] This sense of security, however, did not last long.

In 1931, Gödel proved that mathematics could construct something remarkably similar to the Liar's Paradox. [3] Instead of producing a direct contradiction, Gödel applied self-reference to arithmetic and found a way to assign numerical values to expressions (a trick now known as Gödel numbering). Using this technique, he constructed a mathematical statement that essentially says, "This statement cannot be proved". And here is where things really take a turn. In doing so, Gödel shifted the problem entirely and created something far stranger than a contradiction: he established a crucial distinction between truth and provability. In mathematics, we usually trust that statements are true because we can prove them. However, Gödel showed that within any sufficiently powerful and consistent mathematical system, there will always be true statements that the system itself can never prove.

It is at this point that the system runs into a kind of logical dilemma. If it could prove Gödel's statement, it would end up proving something false and collapse into inconsistency. But if it cannot prove the statement, then the statement is true - yet forever unprovable within the system. Mathematics can avoid contradiction, but it cannot escape limitation.

Gödel's result became known as his First Incompleteness Theorem. It revealed a fundamental structural limit of mathematics – not just some technical difficulty, not a temporary obstacle, not a missing trick, but a boundary of formal reasoning. In that respect, Gödel did not show that mathematics is broken. Instead, he showed that it has an edge. That isn't to say that we have exhausted mathematics, as this couldn't be further from the truth. There are still countless theorems to prove, patterns to uncover, and structures to explore. But no matter how far we push its boundaries, there will always remain truths that lie just beyond formal proof.

His Second Incompleteness Theorem took this idea a step further by showing that a mathematical system cannot prove its own consistency. In other words, mathematics is unable to prove its own reliability. It cannot internally guarantee that it will never produce a contradiction. Instead, any attempt to prove that the system is consistent requires you to step outside of it. [4] In a sense, it's rather like trying (and failing) to describe the exterior of a building from within it. As it seems, even mathematics, often seen as the most certain form of knowledge we have, cannot completely prove itself.

Despite how it may sound, these limits do not mean that mathematics has failed, nor do they make it undependable or weak. Gödel's theorems expanded our understanding of what mathematics is capable of and showed that it is so rich that no single system of rules can capture all of its truths.

And just when it seemed like this kind of limitation belonged only to abstract mathematics, something similar appeared in an entirely different field. Just a few years later, in 1936, mathematician Alan Turing uncovered a similar boundary within computer science. He discovered what is now known as the Halting Problem. The question sounds simple enough: given a computer program and its input, will the program eventually stop running, or continue forever? [5] As it turns out, this problem is undecidable: there exists no universal algorithm that can determine the outcome for all possible programs. The proof works by constructing a program that leads to a contradiction if this universal algorithm were to exist. [6]

This result, alongside Gödel's, suggests that limitation is not a flaw of a particular system, but a fundamental feature of logic itself. Although arising in different contexts, both results expose inherent limits in what their respective systems can achieve. Evidently, some questions resist a definitive answer, not because they are meaningless, but because they lie outside the reach of the system. And perhaps that's why, when pushed far enough, even parents fall back on the ultimate answer: "Because I say so."

This raises a deeper question: if even mathematics, the most precise language we have, cannot fully capture its own truths, what does that suggest about our attempts to understand the world? Perhaps complete certainty is merely an illusion, and the search for knowledge is not a quest we can ever fully complete, but something we can only continue to refine.

To conclude, mathematics did not fail to prove everything; it revealed that proving everything was never possible. What began as a simple paradox in language ultimately reflects a deeper truth: all systems, no matter how developed or precise, encounter limits. In trying to understand itself, mathematics encounters the same kind of self-reference that gave rise to the paradox we began with. The boundaries of mathematics are not signs of failure, but expressions of its depth. They reveal that truth is not confined to what can be formally proven, but extends beyond it. In this sense, the limits of mathematics are not the end of knowledge, but the point at which certainty ends and curiosity begins.

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