

The Magic of Magic Squares

Introduction

We have all played games such as cards, sudoku or ludo in our childhood – be it with friends or family. In birthday parties, the card tricks by magicians seemed reality bending. The YouTuber Jeremy Tan for example, makes the card tricks look effortless. However, a curious mind would look deeper and realize that the answer was simply mathematics. From calculating probability to win a game of cards to solving a magic square, we realize that mathematics is omnipresent. As rightly stated by Galileo Galilei, Mathematics is the language in which God has written the universe. From time immemorial, people always needed mathematics. For example, from 1438 to 1533 CE, Incas used rope knots (Quipu) to count and even today modern applications such as ray-tracing in video games, data analysis and neural network require mathematics.

Mathematics in Games and Puzzles

Mathematics has various applications and uses, especially in the 21st century. I enjoy mathematical puzzles and riddles and so I would like to explore the how mathematics is evident in many games and puzzles as well as video games.

A Magic Square

A magic square, simply put, is a square array of numbers, usually positive integers where the sum of the numbers in each row, each column and both main diagonals is the same. The mathematical study of magic squares deals with its construction, classification and enumeration.

The order of the magic square is the number of integers along one side (n), and the constant sum is called the magic constant.

2	7	6	→15	
9	5	1	→15	
4	3	8	→15	
↙15	↓15	↓15	↓15	↘15

The smallest (non-trivial) magic square possible

Magic squares have been found in many historical artefacts all around. In fact, the third order magic square was known to Chinese mathematicians as early as 190 BCE. The first account of a fourth order magic square occurred in 587 CE in India.

But how do we solve magic squares? Well, if the value of n of a magic square is odd, then:

- Start by placing 1 in the middle of the top row.
- Move up and right diagonally to place the next number.
- If you go out of the square:
Wrap around to the opposite side.
- If the cell is already filled:
Move down one step instead.
- Continue until all numbers are filled.

This is called the **De la Loubere** or **the Siamese Method**. Now, if the value of n is even then we have two types of magic squares, i.e, a doubly even magic square (n is divisible by 4) and singly even magic squares (n is even but not divisible by 4). For solving a doubly even magic square, we use the complementary method where

- Fill the square sequentially with numbers 1 to n^2 (left to right, top to bottom).
- Divide the square into 4×4 blocks mentally.
- In each 4×4 block, mark the cells on both diagonals (forming an “X” pattern).
- Replace each marked number using: $n^2 + 1 - (\text{old number})$
- Leave all unmarked cells unchanged.
- The completed square will have equal sums in all rows, columns, and diagonals.

But if the magic square is a singly even, then we need to divide the magic square into four quadrants of size $\frac{n}{2} \times \frac{n}{2}$ to solve further. To put it into steps,

- Divide the square into four quadrants of size $\frac{n}{2} \times \frac{n}{2}$.
- Construct an odd-order magic square of order $n/2$ using the Siamese method.
- Copy this same square into all four parts.
- In the top-left part, keep the numbers as they are (from 1 to m^2 , where $m = n/2$).
- In the top-right part, add $2m^2$ to each number.
- In the bottom-left part, add $3m^2$ to each number.
- In the bottom-right part, add m^2 to each number.
- Swap the first $m/2$ columns of the left two parts (top-left and bottom-left).
- Swap the last $(m/2 - 1)$ columns of the right two parts (top-right and bottom-right).
- Swap the middle column of the left two parts (top-left and bottom-left).

The Magic Constant

The magic constant is the sum that each row, column, and diagonal in a magic square adds up to. For a square of order n , it is given by the formula $\text{Magic Constant} = n(n^2 + 1) / 2$. This value ensures the balance of the square, and no matter how the numbers are arranged (following the correct method), every row, column, and diagonal will always have this same total.

In conclusion, magic squares beautifully combine logic, patterns, and mathematical structure, making them both engaging and intellectually stimulating. From simple odd-order squares to more complex singly and doubly even ones, each type follows its own method yet leads to the same fascinating result

I hope you enjoyed my exploration of Magic Squares!