The Mathematical Temperament.

Hello,
As the title says, I am going to explain the mathematical temperament. What is it?
First of all, let's acknowledge the fact that mathematics is feared by a lot in school. Some even curse it. Some just say it's a game. Whatever.
But I assure you that there is nothing to fear about it. It is the most pure, honest place with absolutely no hand-waving.
Let me get straight to it. Here no trust works. Here trust has zero value. Only the one who is absolutely straight with life and honest to himself knows pure mathematics.

Lets understand with an example what is actually meant by being straight with life and honest to oneself.

Step 1:
$x(2/m) = 8$
Step 2:
$2/m = 8/x$

How does the 2nd step follow from the 1st step?
Most responses will range from 'Never thought of it',' 'That's the way it is done', 'Teacher said so', etc.
The most common will be 'Because x is multiplied on LHS, we divide it on the RHS.'
Think about this reason. Is it really a reason? I mean how does the fact that x is multiplied on LHS justify that we will divide the RHS by it?
The real reason is we divide both sides by x.
Think about this reason. Is it really a reason? We are saying we have two things that are equal. So we can divide both by x and their equality will persist. Now does that sound correct? Does that feel logical?
Do you see my point? No talk about x. We are using the fundamental concept that if two things are equal, then dividing both by the same entity won't do any harm to their equality.

So in reality there is an intermediate step which is almost always skipped in schools as follows:
$(x(2/m))/x = 8/x$

Once again. While doing this we don't need to care about what we are dividing by. We are dividing by the same thing is all that matters.

So this is the mathematical temperament. Questioning every step until it becomes logically true within you. And only one who is honest to himself and straight with life will be able to question every step.
So please remove all authority-pressure and look at every step carefully.

Let’s take another example.

Do you know that a + b = b + a?

‘This is ridiculous’ might be the first reaction.
Think about it. Do you know that for any a and b, a + b = b + a?

‘Yes.’ might be the answer. But all you know is 1 + 2 = 2 + 1 or other such examples. Right? You don’t know whether in general a + b = b + a. Please think about it carefully. The question is ‘Do you know that addition is commutative prior to actually adding two numbers? And the answer is ‘No we don’t know.’

So how do we use it every time we use it without actually checking? Because we assume it to be true.

And that is an axiom. Are you understanding to what extent one can question in math? There is no limit.

This is the way one understands pure mathematics. Questioning anything and everything.

In fact, mathematicians even ask ‘what is 1?’ How’s that?! When I said there is no limit, I truly meant what I said.

This questioning is the mathematical temperament.

Thankyou,
Yash Phatak