The Mathematics of Geopolitics

This essay was inspired by a conversation I had with my mother in my kitchen. My mother was making dinner one evening and viewed this as the perfect opportunity to ask me (an aspiring mathematician) the most basic of all questions.

I am beyond certain that at one point you have experienced the mental taxation of having to answer this question.

My mother proudly stated, as a humanities savant:

"Why does maths actually matter… what does it actually do?"

I sighed, and in the middle of a poorly worded and thought-out argument/reply, I decided to once and for all answer this question and end this absurd line of questioning for the rest of my domestic life. This essay is a more eloquent articulation of my conversation with my mother.

Times of the past could, at best, be described as barbaric. Nothing was certain; it was one person's will against another's. Mathematics offered a sense of absolutism; it offered a simplification of qualitative matters. The first key figure I will introduce is Muhammad ibn Musa al-Khwarizmi, in reference to the Islamic empires of the 8th–13th centuries.

At this time, many significant Greek mathematicians' works were being translated into Arabic, e.g. the works of Euclid, Archimedes, and Diophantus. This set the foundation for Al-Khwarizmi's work. The mathematical work of the Greeks had a distinct flaw, as Al-Khwarizmi saw it: their framework wasn't very practical and had too many constraints. They restricted combinations, not allowing the mixing of rationals and irrational numbers. Al-Khwarizmi and others allowed irrationals to be treated interchangeably as algebraic objects.

Al-Khwarizmi released his first book, in which he stated it had practical value. The book was based on solving six algebraic equations, all variations of a quadratic equation:

Roots equal to numbers – $bx = c$
Squares equal to numbers – $ax^2 = c$
Squares equal to roots – $ax^2 = bx$
Squares and roots equal to numbers – $ax^2 + bx = c$
Squares and numbers equal to roots – $ax^2 + c = bx$
Roots and numbers equal to squares – $bx + c = ax^2$

Arguably, the most profound part of his work was not merely his classification of the equations, but his systematic geometric approach to solving them. He approached them in a way we now describe as completing the square, expressed through the manipulation of areas: a square and a set of rectangles could be rearranged to form a larger, complete square. By taking half the coefficient of the "roots," squaring it, and adding it to both sides, he transformed a complex relationship into a perfect square that could be directly resolved. This was a repeatable, reliable process which could be executed regardless of the numbers involved, transforming mathematics into a systematic discipline and, most importantly, a useful one.

His notation was incredibly different from today's; he relied on a completely qualitatively driven method of describing everything with words, including numbers. This made it very accessible for widespread utility. An example of how he would have written an equation is: "A square and five roots are equal to thirty units."

He truly systematised algebra, as before it had been applied only to individual problems. He referred to his equations as procedures, introducing incredibly early algorithmic operations. This allowed more complexity in financial systems, enabling more complex loans and setting up the potential for significant enterprise.

Geographically, the Islamic nations were expertly positioned for trade. This mathematical innovation was key for merchants, who could now calculate their expenditure and profits very accurately, making their trades more profitable and viable for a larger market. Procedures and equations could be used to convert currency for trading. This contributed to the creation of one of the largest and most sophisticated medieval trade networks, elements of which still persist today.

Next, we jump forward to the 15th–17th century, where nations, specifically European ones, were beginning to embark on great explorations across the globe. They encountered specific issues: accuracy and precision in navigation. Geometrically, the Earth is not 2D due to its spherical nature. The man who managed to climb this mountain mathematically was Gerardus Mercator. His most significant contribution was the Mercator projection.

Before this idea, sailors followed compasses in a linear fashion. Mercator understood that the assumption that following a compass would lead you in a straight line was incorrect; instead, it would follow a Loxodrome. He was the first to map the world using this model so that lines of longitude and latitude appeared as straight lines.

This allowed sailors and explorers to follow straight lines, called rhumb lines, at constant bearings. The fundamental problem this addressed was the incompatibility between the curved surface of the Earth and the flat surface of a map. Mercator's insight was to construct a projection that preserved angles, stretching the map increasingly towards the poles so that these curved paths became straight lines. This meant that sailors could now travel at constant bearings, reducing the complexity of ocean navigation and creating the façade of a linear journey.

Furthermore, calculating latitude was relatively straightforward using basic trigonometric principles; longitude required further innovation by John Harrison, based on the Earth rotating at 15 degrees per hour. The only issue now was that mathematicians were not sailors. The calculations were tedious and left little room for error. Sailors did not have the capability to perform them easily. John Napier provided a solution: the logarithm, a simplification of advanced computation. Large calculations were required to apply formulas such as the sine and cosine rules, but logarithms, combined with tables, simplified the process significantly.

The pivotal nature of cartography cannot be understated; it truly became an indispensable mathematical language. It enabled the control of land and the creation and maintenance of trade routes. Suddenly, the dynamic of force changed: instead of large armies battling for territory on land, navies became increasingly important.

The discovery of calculus by Isaac Newton and Gottfried Wilhelm Leibniz truly marked the end of the scientific dark ages. Whether framed as an invention or a discovery, it was revolutionary. I would argue that it served as a genesis for world-changing innovation. The development of the calculus of variations by Leonhard Euler and Joseph-Louis Lagrange represented a major advancement beyond basic calculus.

I would define optimisation as minimising the expenditure of a finite quantity. This is what the Euler–Lagrange equation achieves. With a function you want to maximise or minimise, you find stationary points—the lowest or highest possible values. However, in the calculus of variations, you do not know the function; instead, you are trying to find the best one. There are infinitely many possible paths—linear, curved, and so on. To address this, you define a functional (a function of a function), where the input is a function, and the output is a single value related to a quantity such as energy or time.

$J[y] = \int ab\ L(x, y, y')\ dx$

Here, x represents position, y(x) represents the function itself, and y’(x) its rate of change. L represents a quantity or “cost,” and the integral sums contributions along the curve. By making infinitesimal variations to the function and observing whether the functional increases, decreases, or remains unchanged, one can determine whether the function is optimised.

The significance of this innovation cannot be understated. Optimisation often dictates supremacy in industry. This understanding accelerated Britain industrially. James Watt applied these principles to improve steam engines, optimising piston motion and energy efficiency. In this sense, the calculus of variations is truly multidimensional. Using these principles, Britain became and remained a manufacturing giant for centuries, powering colonial expansion through a strong export-driven economy.

When I look back at my life, even at such an early stage, I struggle to understand how my life has unfolded in the way that it has. I am approaching the point at which nearly one-fifth of my life is complete. In some ways, childhood follows a linear trajectory. Yet, even though I have not done anything particularly extraordinary, I feel uneasy when considering the probability of the sequence of events that has led me here. The level of contingency required for me to reach this point seems almost illogical. My understanding of logic has always been as a structured framework of sequential steps—yet when I apply that logic to my own life, I am left confused.

Mathematicians and logicians have grappled with the role of rational thought in probability and outcomes for centuries. In the 1950s came another major development: Game theory. The work of John von Neumann established it as an independent field. Unlike previous mathematics, which often modelled physical phenomena, this described strategic interaction between rational agents.

Mathematically, a game is defined by a set of players, a set of strategies available to each player, and a payoff structure. Unlike calculus, which optimises a single function, game theory involves multiple interacting functions. Outcomes depend not only on your choices, but on the choices of others.

The central concept is that of equilibrium. A Nash equilibrium is a strategic profile in which no player can improve their outcome through individual deviation. Crucially, John Nash proved that every finite game has at least one mixed-strategy equilibrium. Even in highly complex systems, there is a guaranteed structure.

With this guarantee, game theory became incredibly powerful. Countries and businesses could be modelled as players, policies as strategies, and outcomes as payoffs. It brought rationality to decisions of immense consequence—war, escalation, or peace. One of the most famous examples is the Mutually Assured Destruction during the Cold War, where two nuclear superpowers operated under a strategic equilibrium. This demonstrates the profound impact of game theory as one of the most influential mathematical innovations.

After this exploration of game theory, I feel a sense of comfort in understanding chaos. As I enter the next stage of my life—the remaining four-fifths—I realise I must embrace the illogical aspects of logic.

This discussion with my mum and this essay have solidified my desire to become a mathematician. My grandfather always says that knowledge is power, and that I should pursue it to any degree possible. I now say back to him: Maths is power.