The Monty Hall Problem: why do we all get it wrong?

By Beth Holmes

“To acquire knowledge, one must study; but to acquire wisdom, one must observe.”

- Marilyn vos Savant

1. To stick or switch?

Imagine you are on a game show and presented with 3 doors. Behind 2 of the doors there are goats and behind the other is a car. You choose a door and the host opens one of the other doors to reveal a goat. Crucially, the host knows what is behind each door and will always open an unchosen door to reveal a goat and give the contestant the option to switch. He then gives you the option to stick with your original choice or switch to the other unopened door. Should you switch?

This is the Monty Hall problem, named after Monty Hall who hosted the game show Let’s Make a Deal which inspired the question. It was popularized by Marilyn vos Savant in 1990 when it featured in her magazine column ‘Ask Marilyn’. Savant correctly answered the question but many readers believed she was wrong. She received thousands of letters, including many from respected mathematicians with doctorate degrees. Not only did these letters furiously insist that she was wrong, they were often insulting and blatantly misogynistic saying things such as ‘May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?’ and ‘Maybe women look at math problems differently than men.’ To illustrate quite how arrogant these insults are, it should be noted that at the time Savant was listed in the Guiness Book of Records as having the highest recorded IQ.

2. Back to the maths

So what is the answer? If you’re saying to switch, well done! You have a strong understanding of probability or maybe you’ve just heard this question before. But if you’re thinking there’s no point in switching (isn’t there a 50% chance you’ll win the car either way?) you're making the same mistake that most people make when they first encounter the problem.

When you first choose a door, there is a ⅔ chance you will choose a goat. In this scenario the host will be forced to open the door with the other goat behind it as they can't open the door that you have chosen or the door which the car is behind. This means the remaining door will contain the car and you will win by switching so ⅔ of the time switching will lead to winning the car. If this still seems wrong, imagine there are now 100 doors with 99 goats and 1 car. You choose a door and the host opens 98 doors, giving you the choice to stick to your original door or switch to the other unopened door. You switch, right? The chance you initially chose the car is only 1 in 100, while the chance you choose any other door is 99/100. The same concept applies to the original 3 door problem.

3. Let's blame the question

Why is this seemingly simple question of probability so difficult for so many people to get their head around? In part it could be put down to the question being badly explained. The question often fails to specify that the host knows where the car is and will never open that door or that the host will open a door and give the contestant a chance to switch regardless of whether they originally choose the car or goat. While most people make these assumptions anyway (the game wouldn't work very well if the host revealed where the car is to the contestant!) they can change the question's answer.

There are several variations of the Monty Hall Problem such as the 'Ignorant Monty' variation in which the host randomly chooses which door to reveal from the 2 that the contestant did not choose. If the host reveals a goat and then gives the contestant the option to switch, the contestant actually has a 50% chance of winning the car whether they change doors or not. If you've only just wrapped your head around the answer to the original question this might be your last straw. Like in the original, there is a ⅔ chance you originally pick the goat. There is then a ½ chance that the host will reveal the car and ½ chance that the host will reveal the other goat. This means there is a ⅓ chance that you chose a goat and then the host reveals the other goat and a ⅓ chance that you chose a goat and the host reveals a car. In the ⅓ chance that you initially chose the car, the host will reveal one of the goats. When the host reveals a goat, it is equally likely that your initial choice was a goat or a car.

4. Are we all just bad at probability?

However even when the question is unambiguous, as I made sure it was in the first section of this essay, many people still get the question wrong and cannot see why they are wrong. Put bluntly, the problem is that humans are naturally not very good at probability. There are many examples of results in probability which just don't 'feel right' to us such as the fact that there only needs to be 23 people in a room for it to be probable that at least 2 will share a birthday. Take the example of flipping a coin and getting heads 99 times in a row. If it is flipped again there is a 50% chance it will come up heads again but even when you're aware of this it 'feels like' it's more likely to come up tails. This is because we realise that flipping heads 100 times in a row is incredibly unlikely (1 in approximately 1.268×10^{30} if you're wondering) but struggle to see the 100th coin flip as an independent event. Similarly, certain orders of playing cards feel 'more random' even though in a fully randomised deck of cards, any order of cards coming up is equally likely.

Equiprobability bias is the tendency to assume that all outcomes are equally probable. This seems like a stupid mistake to make. If I try to talk to my cat, I could say that there are 2 possible outcomes because he will either start talking or he will not but those outcomes are obviously not equally probable! However equiprobability bias is likely the reason so many of us get the Monty Hall problem incorrect. At first we see there are 3 doors so we assume (correctly) that there is a ⅓ chance we have selected the car, so when one door is opened we assume there must now be a ½ chance we have selected the car. We assume that each door is equally likely to be in front of the car. This ignores that the host has given us new information in their choice of which door to open. When the problem is expanded to more doors, such as the earlier example of 100 doors, most people will be able to intuitively understand that they should switch but some will struggle to apply the same logic to the original question. This shows how hard it is to comprehend probability when it goes against our intuition.

5. How to be smarter than pigeons

Studies have found that when people are presented with a simulation of the Monty Hall problem, not only do most initially stick, but even after hundreds of times running the simulation not everyone switches. In contrast, pigeons quickly learn that switching is the optimal strategy. Now I'm not implying that this shows that pigeons have a better grasp of probability than humans but they do have an advantage over humans; they aren't held back by their prior assumptions. I love the Monty Hall problem not only because it is mathematically interesting but it also teaches us something that is important in both maths and in life; to accept that we can be wrong even when it seems absolutely impossible.

Sources

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