The ***SAD*** reality of our inability to count grains of sand, an essay about ***combinatorics***.

(Hi! Thanks for reading! It's intended to be super light-hearted. Please ***enjoy***.)

OPENING PARAGRAPH:

You're competitive. (*Nice*!)
What do competitive people do?
CORRECT!
We (*yes, we*) joined kids' mathematics games to boost our ego!
So, did *we* win? …*no*.
Did *we* place dead last? ***YEAH!***
But it's okay. Life isn't all about mathematics anyway.
Or is it…?

Realistic scenario: You meet a genie. It says that it will give you infinite money if you say something no one has ever said before and no one will ever say in the future, if you fail, you die. You're smart, and greedy (*and competitive too… though I think I said that already…*), so you take a **deck of cards**, **shuffle them** and say them in their **order**.
Sorry! you just died, but hey! The **REAL** question: What were the odds? (*Let's try to explain this… later though…*)

1. INTRODUCTION.

1.1: ORIGINS:

Let's take you to Greece! You're a philosopher, Xenocrates. (*Wow, you're a lot of things…)*.
You want to know how many syllables the Greek language has. Good, it's a first attempt at a difficult problem in permutations and combinations.
The answer? Around **1002000000000**. (*No, I didn't make that up just to be dramatic… haha...*)

Our next stop: India! There are six tastes: **sweet, pungent, astringent, sour, salt and bitter**. In how many ways can you combine them?
It's okay, you don't have to answer. Bhagavati Sutra already did it for us.

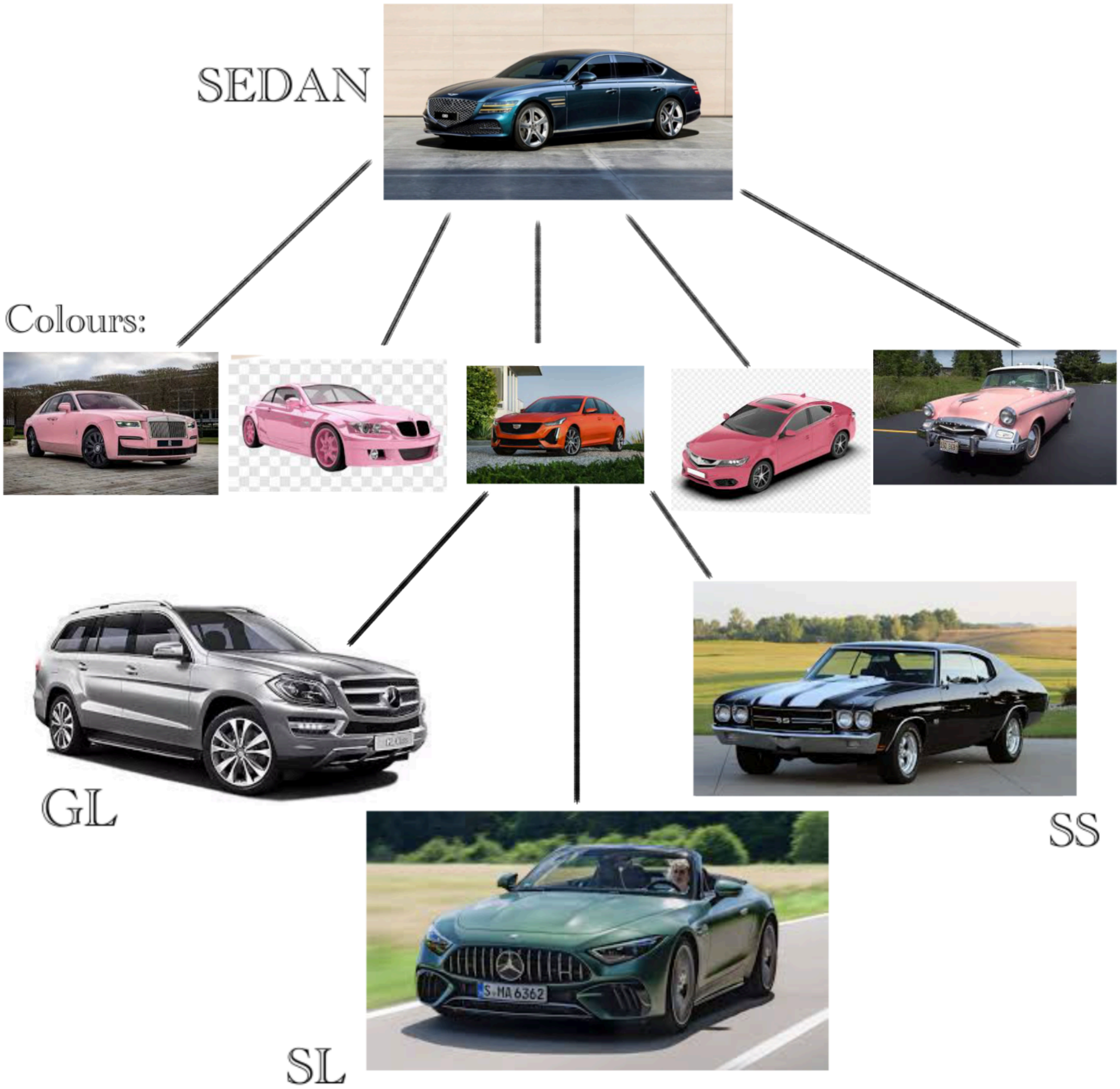
China is next, but since you must be tired from all this travelling (*me too*), it's our last stop.
Do you know what a **hexagram** is?
If you don't, that's fine. I just found out myself. It's just a figure made of six stacked lines, either broken or straight.
So how many are possible? **2^6** or **64.**

Now… how did we do that?

2. HOW DO WE COUNT AND WHAT IS A FACTORIAL?

2.1: COUNTING:

I hope you like cars, we're going shopping. (*Weren't we tired…?*) There are two types of cars here; sedans and hatchbacks. We have five colours; pale pink, baby pink, deep pink, bubblegum pink and ***ORANGE***. There are three models; a **GL**, **SS**, and an **SL**.
How many options do we have?



This and the same goes for hatch backs!

You would multiply all of them together because they are **dependant** on each other, and say 30 (2*5*3). But no, we ***DON'T*** want an orange car. It's 24 (2*4*3).

Now what if the options are **independent**?
It's okay, just add them up instead. For example, 12 options for the sedans (1*4*3) and 12 for hatchbacks (1*4*3), *Voila*! 12+12 = 24.
This is the **fundamental principle of counting**!

2.2: FACTORIAL*!* (*Pun intended*)

You have a whole number. Now, unless it's not **zero** (*I'm foreshadowing)*, there must be a natural number, either, before it or it itself. Multiply all of them together.
1: 1
2: 1 x 2= 2
3: 1 x 2 x 3 = 6…

Seems cool. Shouldn't it have a denotation?
Lucky you, it does! For a number n, n **factorial= n!**

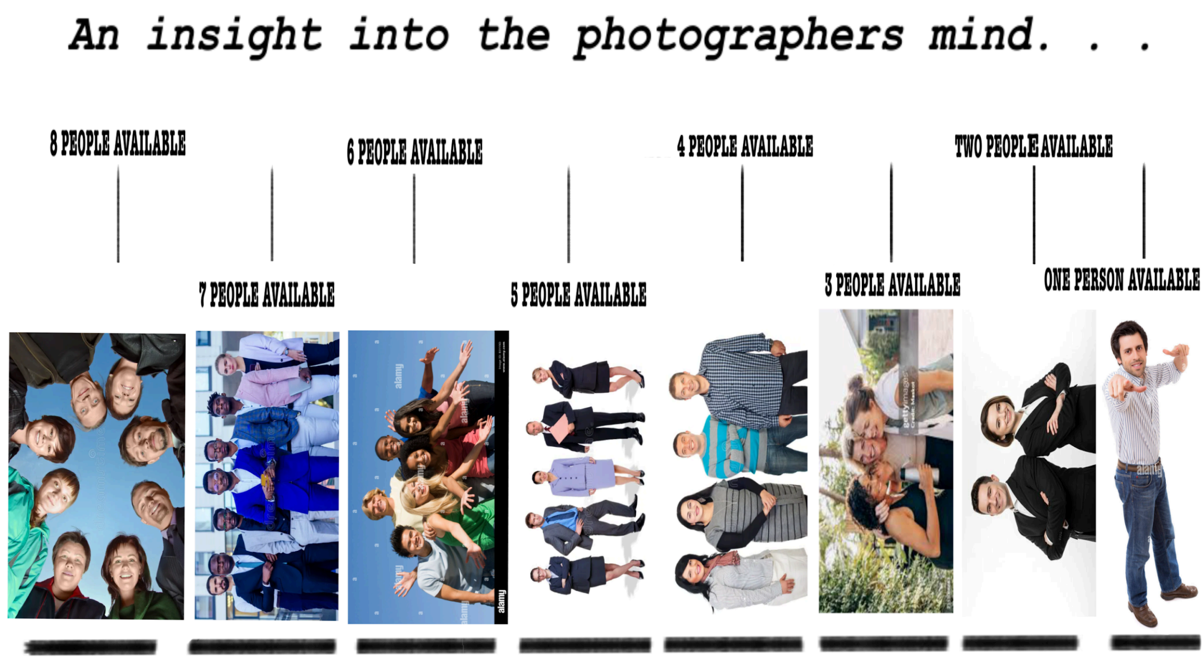
Hmm… do you see it? **A pattern!**
1! = 1
2! = 1! x 2
3! = 2! x 3

lets generalise this: **n! = (n-1)! x n**
You're ***also*** curious, so you put 1 instead of n, because (1-1) is 0. No, fortunately, you didn't defy the laws of the universe, you're safe.
But hey! You just discovered a new convention we will use.
You got: 1! = 0! x 1 => **0! = 1.**
Nice work!

2.3: QUESTIONS:
But, seriously, when will this even help you?
Actually realistic scenario:
You're at a dance party with your best friend. *Hi!* I'm there too, with my best friend.
Is that all? **No**.
We have two more sets of friends. (*Don't know who though, maybe you do?*) We are seated in a line, for a photograph. How many ways in which the photographer can arrange us?

You would think, 8 people, so, there is an **available** 8 that can sit on the extreme right/left. One spot is filled so, 7 available next to them, then six, then five, so on.
So, 8 factorial?

Or simply put: **8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 or 8!**



But honestly, no one wants to get separated from their best friend, yeah?
So it's **4! x (2!)^4**

How?
We think of two people as one unit, so four units.
That is **4!**, But someone sitting on the left is different from them sitting on the right, so that makes 2! possibilities for all groups, so **4! x (2!)^4** in total.

4 3 2 1 <—— No. of options

unit1 unit2 unit3 unit4

So, **4 x 3 x 2 x 1 = 4!**

for each unit:
__ __ **<—— 2 spots, same thinking.**
That's **2!**, but for **four units**!

3. PERMUTATION AND COMBINATIONS.

3.1: DEFINITIONS:

We did a few operations, but what does **permutations** *OR* **combinations** even mean? (*My bad*).

Let's say you have a group of N number of distinct items and you arrange them in groups of numbers less than or equal to N.
Now, either you took **their order** into consideration or you didn't (*maybe you like set theory…? Get it?*). The former is permutations and the latter is combinations. Simple? Not really? Okay, let's clarify.

3.2: QUESTIONS:
I designed a building for this purpose (*yeah, you're always welcome*). It's a three story. It has four rooms on the ground floor, and two on the first and second. It's for six people. We don't share rooms (*effects of lockdown*). How many ways in which we can stay there?

We have eight rooms, and six people. Now, everyone is different. So? Yep! The order matters. We will use **permutations** then. 6 people to be arranged into 8 rooms. How do we write that simply? I got you. $^{8}\mathbf{P}_{6}$ or **P(8,6)**. (*Fancy!*)

Is that all?
No, we **DON'T** leave floors empty. What's the point of having them then? Let's subtract the possible number of values. If the first floor remains empty, how many ways to arrange? **6!** (P[6,6]). What about the second? **6!**
Good work!
It becomes?
$^{8}\mathbf{P}_{6}$ **- 2*6!**
"But what's the numerical value!?"
here, calculate it yourself: ***P*(n,r) = n!/(n-r)!**, because you're competitive.
Just kidding, it's **18,720**. (*Trust me.*)

Let's do another question. Exciting. (*For me at least*)

Do you like divisibility tests?
Let's take you back to elementary. Any number ending with a zero/ five is divisible by 5.

How many numbers between 3000 and 4000, made from only the digits 3, 4, 5, 6, 7, 8 are divisible by 5? No repetition of digits!
Let's fix somethings, yeah?
General from of our number: **3 — — 5**, it's given!
So 2 spaces and 4 choices. I know, you got it.
P(4,2). Yep! 12.

"What about the genie?" you ***might have*** asked if you were still here with us… (*or did you forget that, huh? Maybe you **aren't** greedy after all.*)

Think of 52 chairs and the ***guards* of the queen** from *Alice in Wonderland*! (*They are cards, if you didn't know…*)
So, just like the photography problem, it's **52 x 51 x … x 20 x 19 x … x 2 x 1.**
Yep, **52!**
So what were the odds? Yes, **1 in 52!**
52 factorial? *Well it's a hundred billion*. Just kidding, it's **way more** than that.
It's not comprehensible but it's **8.06 x 10^67**. (*I'm actually not joking.*)

3.3: FORMULAS:

I don't wanna disappoint you, but it's getting difficult to proceed without formulas, yeah?
N would be our total, R is the number of items we want to arrange.
Combinations: n!/(n-r!)(r!) = C(n,r)
Permutations: n!/(n-r)! = P(n,r)

Now, it would have been your turn to ask questions, but I know I told you to ***enjoy***, so I'll ask and answer myself. (***?***)

What if the objects are arranged in a **circle**?
N becomes (n-1).
Why? Here, an example:
1,2,3,4 is the same as 2,3,4,1, which is the same as 3,4,2,1 and lastly, the same as 4,1,2,3.

What if anti clock wise and clock wise rotations are the considered the same?
No worries, just do, **total / 2**.

Can this be used in **geometry**? Good question… quite interesting…

Do you like pentagons? No?
(They look like a house sometimes, but you're not Dorothy, so I get it.)

How many diagonals does it have?
5 vertices, let's name them: **A, B, C, D, E**. Let's start at A, for 'A house' *(lol)*.
We can connect it to two opposite vertices for **2 diagonals**. Then from the two of its adjacent vertices, four more diagonals.
Or is it?
They have one **common** diagonal. So it's actually **three more**. That's all of them. **5 diagonals**.

But what does that give us? We could have done that for any vertex. So that's **C(5,2)**.
Yes, finally! ***Combinations!*** It doesn't matter which vertex you start, they all have two opposite vertices.

Why 2? That's the number of non adjacent vertices here, or in general, **N-3**.

Why subtract 3? To include the vertex itself and the adjacent sides.

But how do we account for the over lapping diagonals we count?
Yep! They are equal to 5, the **number of sides**.

Now what do we like? Competition, you might say?
No… I meant like patterns… let's GENERALISE!

Number of diagonals = n (n-3)/2.

What…?

It's $C(n,2) - n$, so $[n!/(n-2)!2!] - n$.
So, $[n(n-1)/2] - n = n(n-3)/2$.

4. PIGEON HOLE PRINCIPLE.

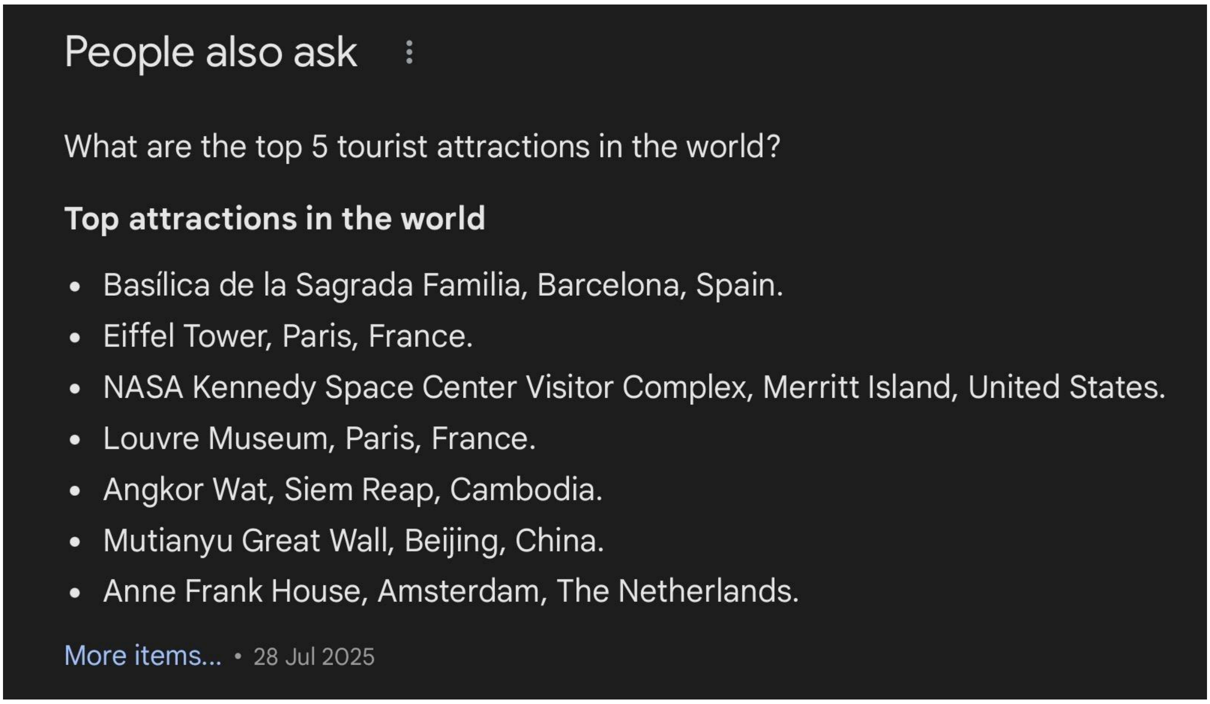
4.1: DEFINITION:

Bye, the lesson is almost over. I'm leaving so I have to pack three of my things. You have given me two boxes. (*Not nice!*) So, ***one box must contain two things.***
I forgive you, for you have just discovered the pigeon hole principle.
(*Yes that's it,* ***n pigeons, m holes, $m<n$, so one hole has more than one pigeon***)

Sounds like common sense, right?
The hard part is ***identifying*** the pigeons and the pigeon holes.

4.2: QUESTIONS:

We started with travels, let's end with travels.
"*But hey! I'm indecisive.*" I got you. (*That's not exactly me…<insert laughter>*)



Spain, France, USA, Cambodia, and then China**.**
Can you prove at least two of them lie in the same hemisphere?
And nope, I'm **not** talking about the northern or southern hemisphere.
It could be a division anywhere! (*As long as it splits the globe into two…*)

Let's **create** our pigeons and pigeon holes. (*That's poetic.*)
Draw a diameter passing through any and only two of the places.
Do you see it? **Yes!**
Three places and two hemispheres! So one hemisphere must have more than one place!

Did you know?
Four is my favourite number.
If you take a group of 5 numbers, I'm sure there is at least one combination of two numbers, such that, if you subtract them you're gonna get a number divisible by four.

(*Yeah this actually works for every number (n) and (n+1)* ...***never mind***)

So what happens when you divide a number by 4?
The **remainders can be 0, 1, 2 and 3**. But when we take five numbers?
Yes! One of these numbers has to repeat! So, when you subtract them, the **remainders get eliminated** and the number becomes divisible by four!

CONCLUSION:

The infinite monkey theorem states that a monkey hitting keys independently at a random typewriter keyboard for an infinite amount of time will surely type any given text, including the complete works of William Shakespeare. ***But hey! Now that you can use permutations and combinations, it is your turn to calculate what's the probability of that monkey writing down my essay.***