The Sulba Sutras: A Vedic Approach to Approximation

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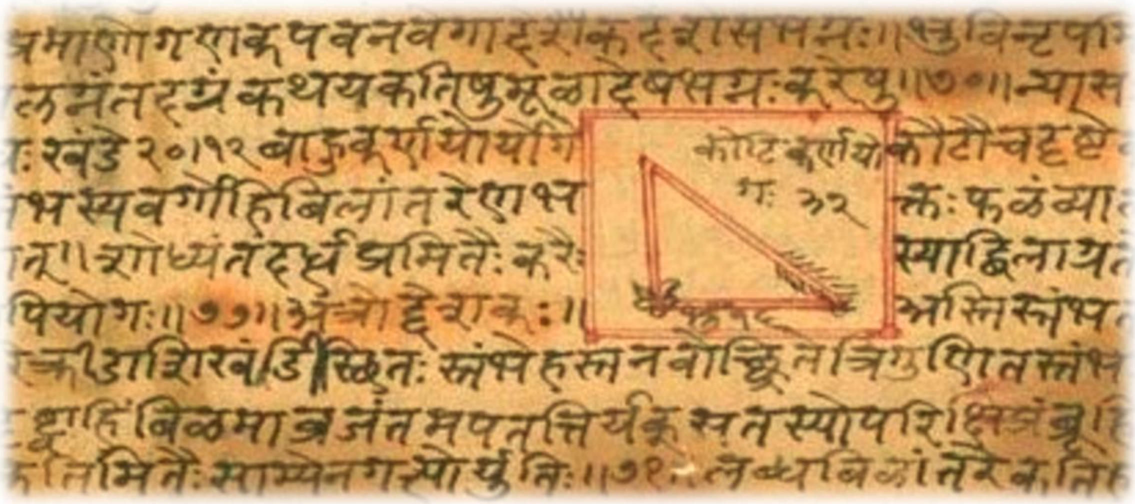


1) Introduction

When one says “mathematics”, one often pictures large calculations, pages of steps, and the occasional dilemma of how many candies Sam has left after sharing with his friends. In such problems, textbooks usually lead us to clean exact answers-like 1 or 0. This often gives the impression that mathematics is a domain of certainty, where every answer is "simple". In practice, however, many of the most fundamental quantities in mathematics resist such exactness. Instead, we approximate them, trading precision for practicality.

2) Why do we need approximations?

Approximation becomes necessary for several reasons.
Many fundamental quantities, such as $\sqrt{2}$ or π, cannot be expressed as simple ratios. For example, π is often approximated to the fraction 22/7 for ease of calculation.
Furthermore, all physical measurements are ultimately limited in precision, making approximate values both necessary and unavoidable.
In geometry and trigonometry in particular, simple constructions often give rise to irrational numbers. For example, the altitude of an equilateral triangle has a value $\sqrt{3}/2$ times its side length.
This need for approximation is not a modern development; long before algebraic techniques and decimal expansions, ancient mathematicians developed their own methods to estimate such quantities. Approximation helps us make quicker decisions by giving practical answers to complicated problems—especially when getting the exact value is not possible. For example, in machine learning, approximation algorithms are important because the exact answer would take too long to compute.

3) What are the Sulba Sutras?:

The Sulba Sutras are part of the larger group of texts called the Shrauta Sutras, considered to be appendices to the Vedas. They are the only sources of knowledge of Indian mathematics from the Vedic period.
Unique Vedi(fire-altars) shapes were associated with unique gifts from the Gods and were geometrically built using rules prescribed in these Sutras. In these texts, mathematics emerges not from theory, but from necessity- the act of building, measuring, and transforming space with nothing more than a rope.

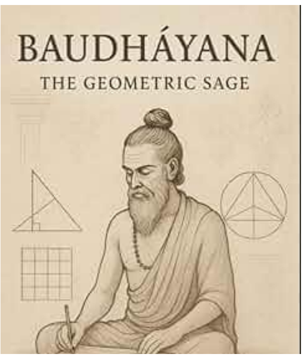
Figure:1

Falcon-shaped *vedi* excavated from Purola, Uttarkashi; likely belonging to the Kuninda period (150 BCE - 250 CE).

4) More About the Sulba Sutras:

A distinctive feature of these Sutras is that it represents a culmination of the works of ancient scholars from the Later Vedic period. Among the contributors to this guide were Baudhayana (c. 800 BCE), Apastamba (c. 600 BCE) , Katyayana (c. 300 BCE) and Manava (c. 750 BCE).

The name Sulba Sutras (or *Śulbasūtras*) is derived from two Sanskrit words : Sulba meaning a "cord," "rope," or "measuring string" and Sutra-Meaning a "thread" or "aphorism."
Essentially, "Sulba Sutras" translates to **"The Aphorisms of the Measuring Cord."**
The Sulba Sutras describe a wide range of mathematical concepts long before they were formalized in the West. These include the Pythagoras theorem, methods for calculating irrational numbers, Solving linear equations through geometric construction and techniques to transform shapes while preserving their area.



What began as a set of ritual instructions unfolded a mathematical framework consisting of approximations and transformations way before its time.

5)The approximation of √2:

One such example is √2, a number that cannot be expressed exactly as a ratio of integers. A classical proof of this uses contradiction: assume √2 can be written as a fraction of two coprime integers. Squaring both sides leads to the conclusion that both numerator and denominator must be even, contradicting the assumption that they are coprime. The contradiction is unavoidable, and so is the conclusion—√2 is irrational. Geometrically, √2 can be obtained as the diagonal of a unit square, or equivalently, as the hypotenuse of a 1×1 right isosceles triangle.
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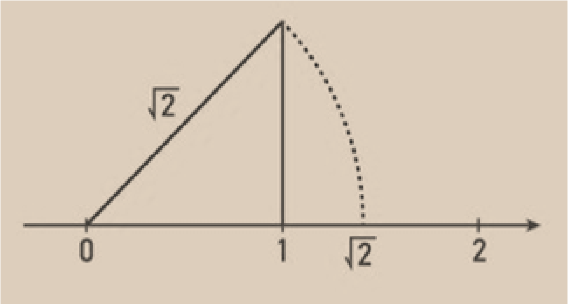


Figure 2: Construction of √2 using a unit triangle

The method for approximating √2 is described in verses I.61–2 of Baudhayana's Śulba Sūtras, and is later explained in greater detail in section I.6 of Apastamba's work. Interestingly, the structure of the Śulba approximation for √2 resembles a truncated series expansion (same as Taylors series developed in 1715).
The Baudhayana Sūtra describes a method for calculating the value of √2 as follows:

समस्य द्विकर्णि प्रमाणं तृतीयेन वर्धयेत।
तच् चतुर्थेनात्मचतुस्त्रिंशोनेन सविशेषः ।।

Transliteration:

samasya dvikaraṇī. pramāṇaṃ tṛtīyena vardhayettac caturthenātmacatustriṃśonena saviśeṣaḥ
1)Sama – Square;
2)Dvikarani – Diagonal (that which divides the square into 2)
3)Pramanam – Unit measure
4)tṛtīyena vardhayet – increased by a third;
5)Tat caturtena (vardhayet) – that itself increased by a fourth
6)Caturtrimsah savisesah – is in excess of 34th part.

This verse provides a step-wise approximation:
Begin with taking the side, add one-third of it, then add one-fourth of that third, and finally subtract a small correction. Applying this concept to a unit square gives us the approximate value of √2. While the verse describes the approximation algorithmically, its geometric execution reveals an even deeper mathematical understanding .

Figure 3:
Area Transformations

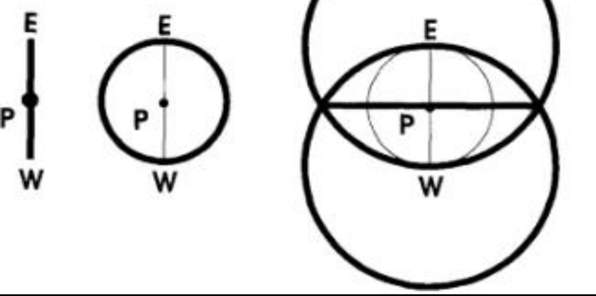


Figure 4:
Area transformation using differences of areas

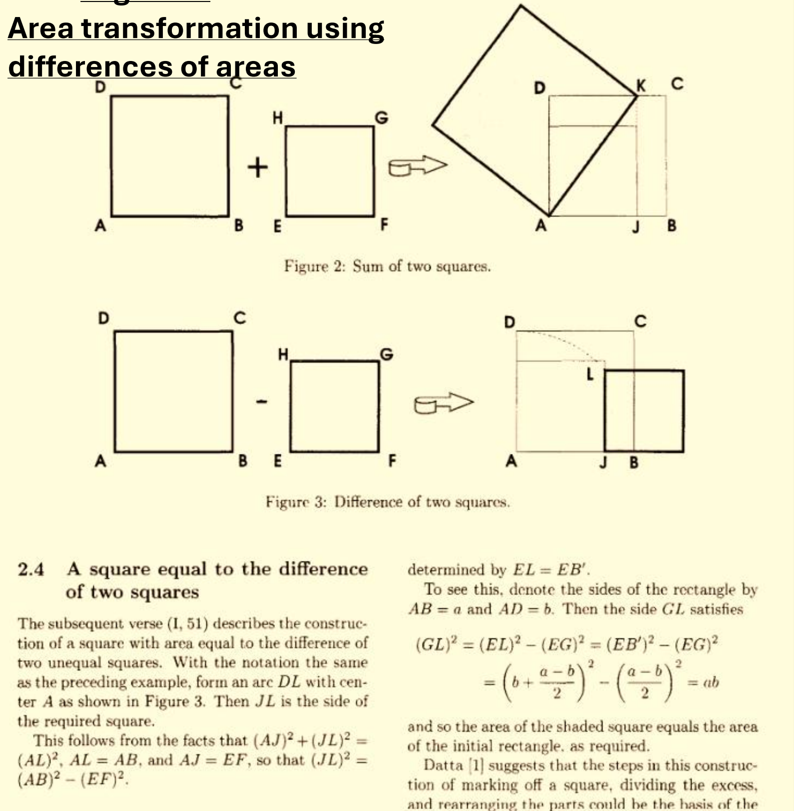


Figure 2: Sum of two squares.

Figure 3: Difference of two squares.

2.4 A square equal to the difference of two squares

The subsequent verse (I, 51) describes the construction of a square with area equal to the difference of two unequal squares. With the notation the same as the preceding example, form an arc DL with center A as shown in Figure 3. Then JL is the side of the required square.

This follows from the facts that $(AJ)^2 + (JL)^2 = (AL)^2$, $AL = AB$, and $AJ = EF$, so that $(JL)^2 = (AB)^2 - (EF)^2$.

determined by $EL = EB'$.

To see this, denote the sides of the rectangle by $AB = a$ and $AD = b$. Then the side GL satisfies

$$(GL)^2 = (EL)^2 - (EG)^2 = (EB')^2 - (EG)^2$$
$$= \left(b + \frac{a-b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

and so the area of the shaded square equals the area of the initial rectangle. as required.

Datta [1] suggests that the steps in this construction of marking off a square, dividing the excess, and rearranging the parts could be the basis of the

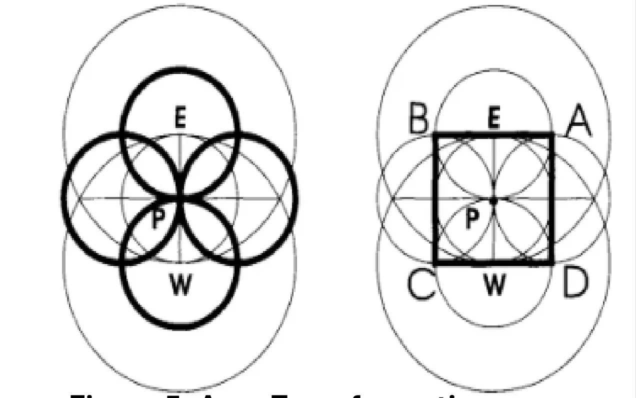


Figure 5: Area Transformations

6) Executing the approximation method:

The elegance of this method becomes most apparent when interpreted geometrically.

STEP 1:Let us consider 2 squares PQTU and TURS, each of side length 1. Square TURS is divided vertically divided into 3 strips, each of dimensions 1x1/3.

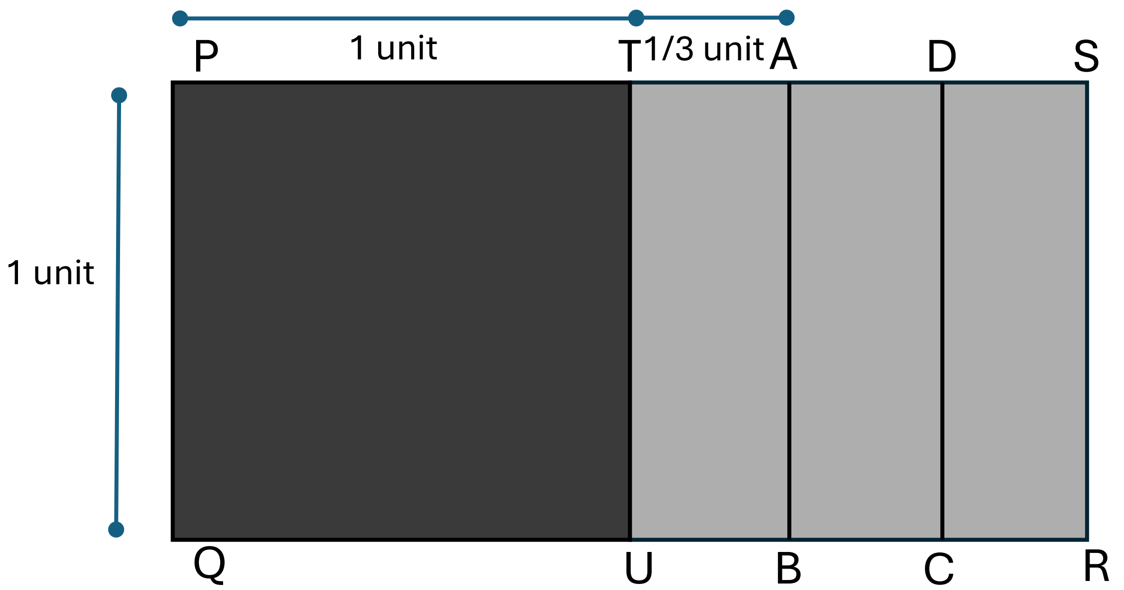


Figure 6:Two unit squares are considered; one is divided into three equal vertical strips.

STEP 2:Now, 2 pieces of the second square are placed above the first square to form an approximate square of side 1+1/3, with a missing square of 1/3x1/3 dimensions

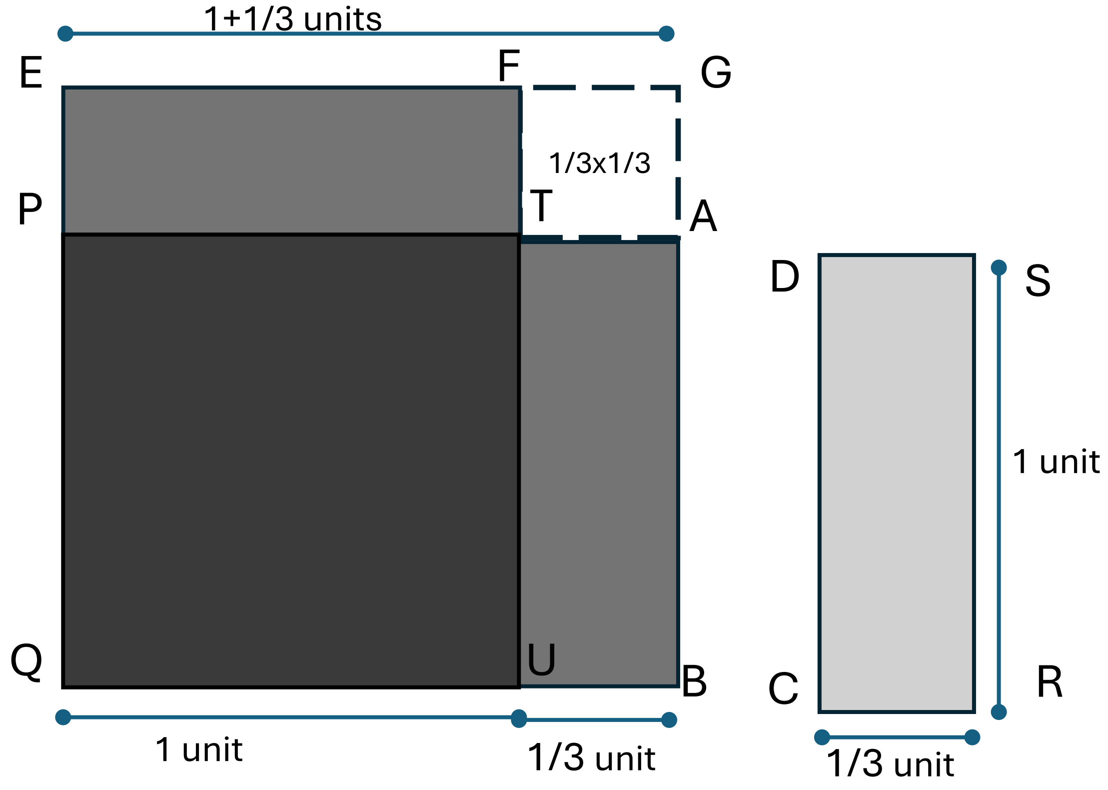


Figure 7: Addition of areas to the unit square

STEP 3:The 1/3x1/3 square if filled by taking a square of the same area from the remaining strip DCSR, thereby completing a square of length 1+1/3 units

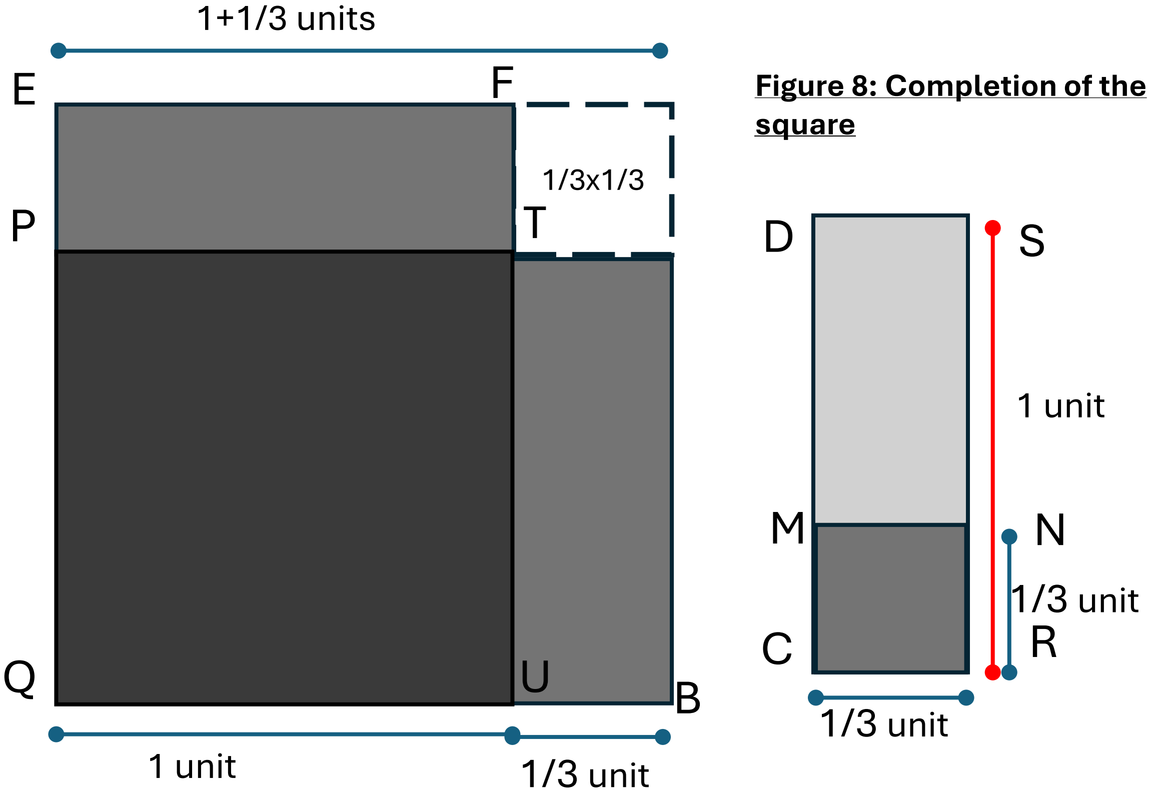


Figure 8: Completion of the square

STEP 4:Finally, the remaining portion of strip DMNS is divided equally into small units, and placed around the square. This contributes an additional area equivalent to $\frac{1}{12}$,refining the approximation

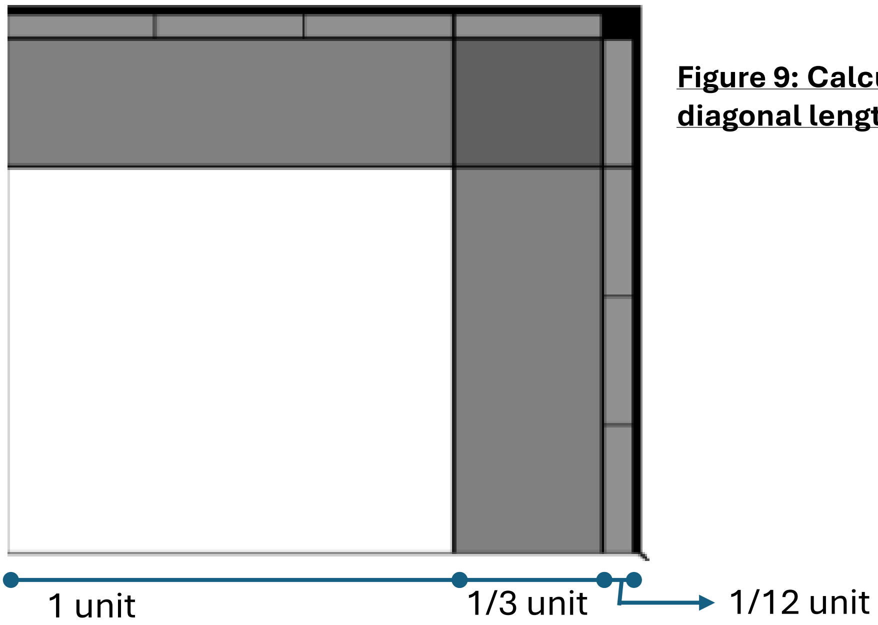
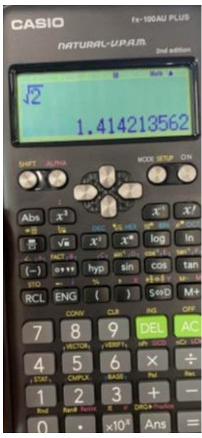


Figure 9: Calculation of diagonal length

The black portion of the diagram signifies 34th part of the previous amount subtracted(1/34x1/12), that is removed from the square.



Putting the above approximation together mathematically:

$$\sqrt{2} = 1 + 1/3 + 1/4(1/3) - 1/34[1/4(1/3)]$$

$$= 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} = \frac{577}{408}$$

$$\approx 1.4142$$

The approximation $\frac{577}{408}$ that differs from $\sqrt{2}$ by less than 0.0002 ,was achieved over 2,500 years ago—without algebra, calculus, or decimal notation

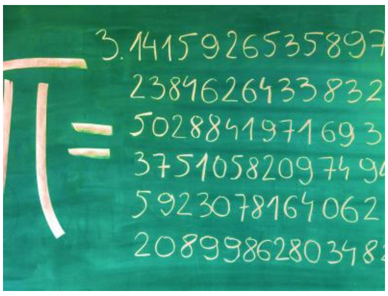
7) Approximation of π

Another approximation in the Sulba sutras is the approximation of π. The primary shloka (verse) for transforming a square into a circle comes from the Baudhayana Sulba Sutra (1.58). It is written in Sanskrit and describes the geometric "rope" (Sutra) technique.

चतुरश्रं मण्डलं चिकीर्षन् साक्ष्ण्यार्धं मध्यात्प्राचीमभ्यापातयेत् ।
यदतिशिष्यते तस्य सह तृतीयेन मण्डलं परिलिखेत् ॥

Transliteration:

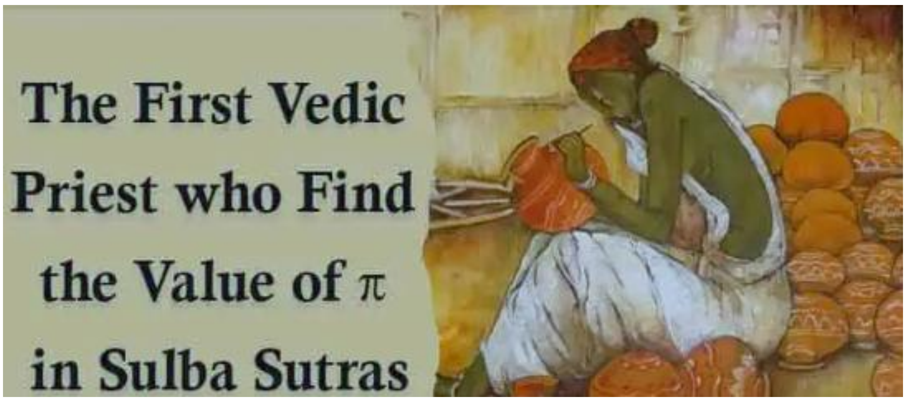
Caturaśraṃ maṇḍalaṃ cikīrṣan sākṣṇyārdhaṃ madhyāt prācīm abhyāpātayet | Yadatiśiṣyate tasya saha tṛtīyena maṇḍalaṃ parilikhet ||



- **Caturaśraṃ:** A square.
- **Maṇḍalaṃ:** A circle.
- **Cikīrṣan:** Desiring to make (to transform).
- **Sākṣṇyārdhaṃ:** Half of the diagonal.
- **Madhyāt:** From the center.
- **Prācīm:** Eastward (or toward the side/boundary).
- **Abhyāpātayet:** Let it fall or swing down.
- **Yad-atiśiṣyate:** That which exceeds (the portion of the diagonal that stic
- **Saha tṛtīyena:** Together with a third (part of that excess).
- **Parilikhet:** Describe or draw (the circle).

The verse instructs you to take half the diagonal of the square and pivot it from the center toward the side. Because the half-diagonal is longer than the distance to the side, a portion will stick out. You are then told to take the part that stays within the square (the half-side) and add one-third of the part that sticks out. This combined length becomes the radius of your circle.

From the approximated radius, we can divide the area of the square and the square of the radius of the circle to obtain the approximated value of π

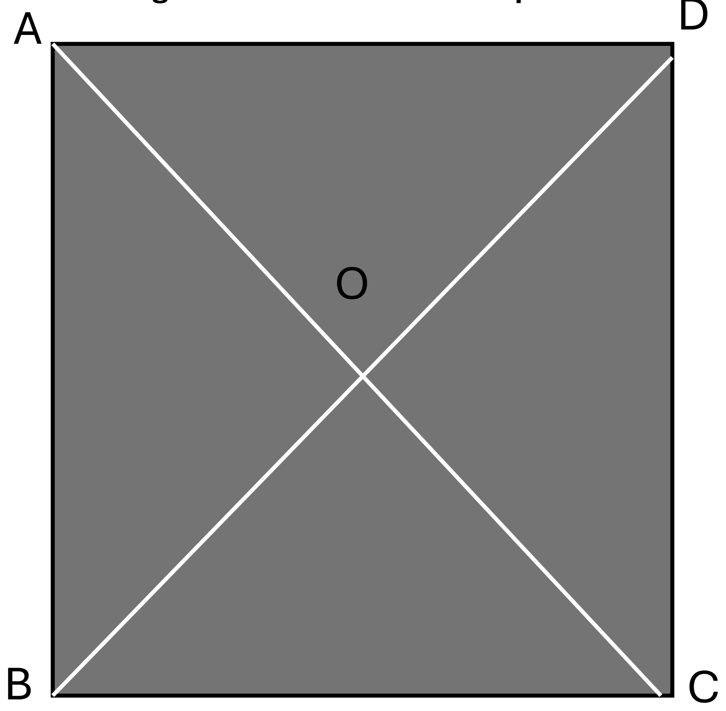


8) Executing the approximation method:

To transform a square into a circle with an approximately equal area, the Sulba Sutras provide a step-by-step method based on the diagonal of the square.

STEP 1: Let ABCD be the square we wish to transform. We find the center point O by intersecting the diagonals.

Figure 10: Division of the square



STEP 2:Now, Let OE be half the length of a side (the distance from the center to the midpoint of a side) and left OF be half the length the length of the diagonal(distance from the center to a corner).

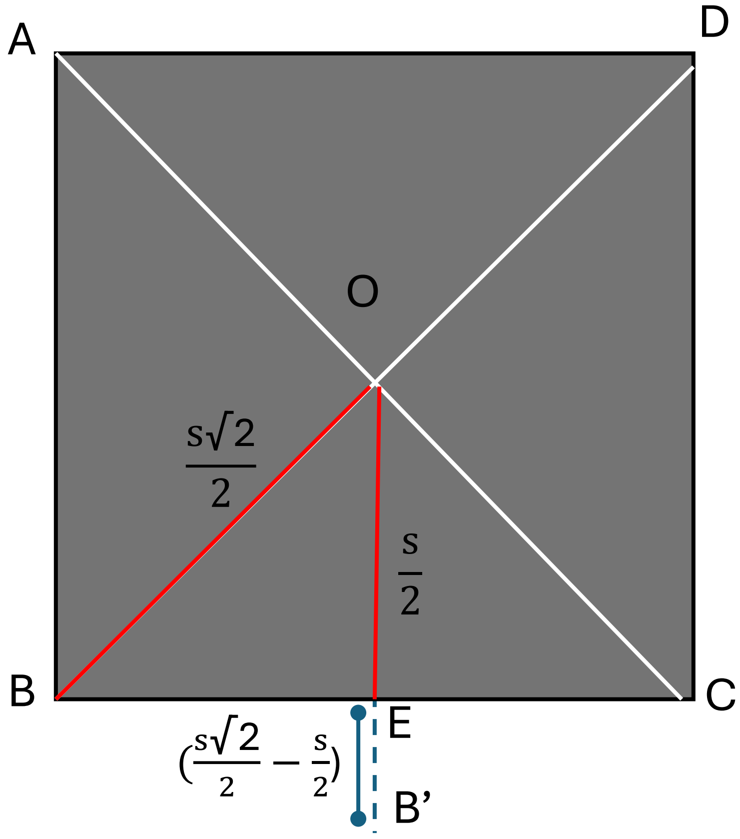


Figure 11:Regarrangement of lengths

STEP 3: The radius of the new circle is not simply the distance to the side or the corner. Instead, it is found by taking the distance to the side(OE) and adding 1/3 rd of the difference between the half diagonal(OB) and the half side(OE)

Adding all the terms together, we get :

$$r = \frac{s}{2} + \frac{1}{3}\left(\frac{s\sqrt{2}}{2} - \frac{s}{2}\right)$$

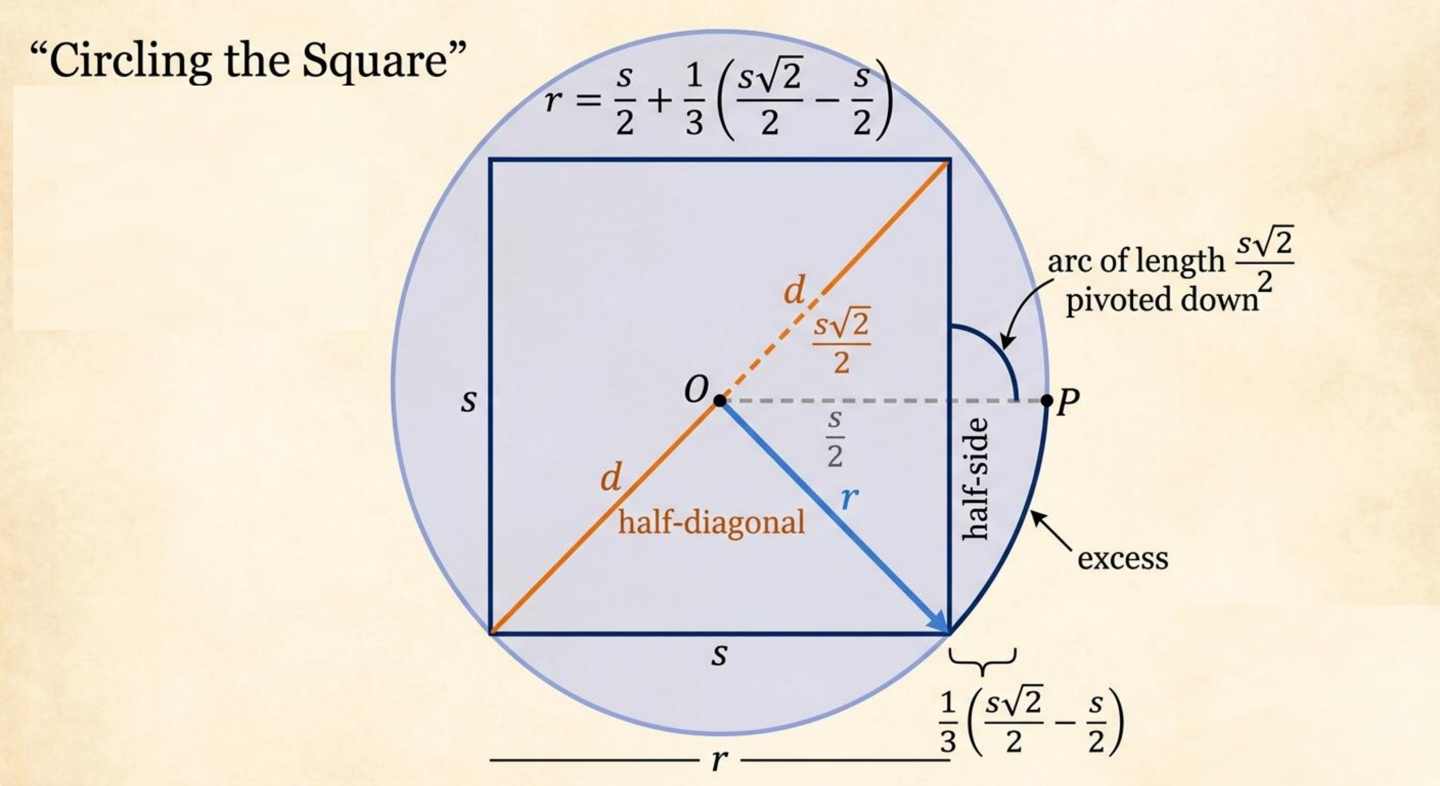


Figure 12: Complete diagram for calculating π

Using a unit square for ease of calculation,
we get radius of the circle using that formula to be
r=0.5+0.06903=0.56903
As this approximation is based on conservation of areas, both the area of the unit square and the circle must be the same.
This gives $\pi r^2=1$ or π as 3.088
This value is only 1.7% off from the actual value of π

For a bricklayer in 800 BCE using a rope, this was an incredibly accurate way to ensure the ritual area remained constant across different geometric shapes.

9)Two Worlds of Approximation: Algebra and Geometry

Approximation in mathematics has evolved along different lines. In modern mathematics, the most common approximation tool used is the Taylor series, which expresses functions as an infinite sum of polynomials.

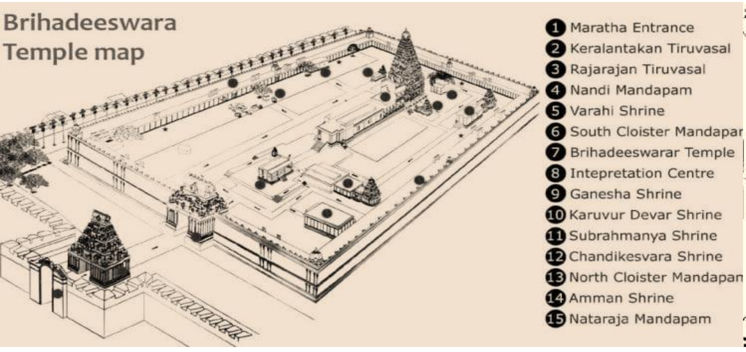
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \qquad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

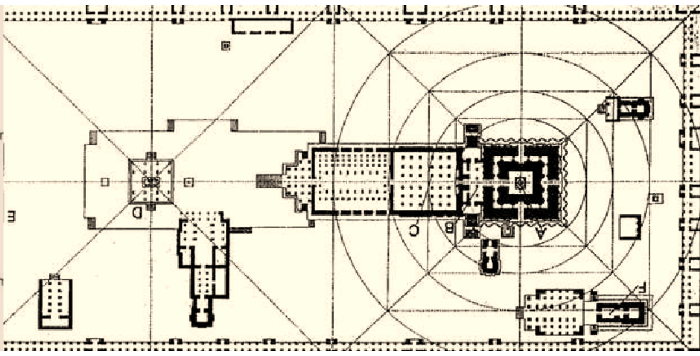
However, such methods rely on advanced symbolic manipulation and calculus—tools that were not available in ancient times. This makes the methods found in the Śulba Sūtras even more remarkable, as they achieve accurate approximations using purely geometric reasoning.

10) Application of the Sulba Sutras in ancient constructions.

The Sulba Sutras reveal that sophisticated mathematical approximations and calculations existed long before algebra and calculus were formally documented. Through simple geometric constructions and transformations, ancient scholars could approximate many non-terminating numbers with remarkable precision, driven not by abstraction, but by a necessity. These sutras not only facilitated the construction of giant temples and altars but were instrumental in building out entire cities of the ancient civilization that continue to stand the test of time.

Figure 13:Geometry of the Brihadesvara temple





To understand the Brihadisvara Temple(constructed in 1010 CE) through the lens of the Sulba Sutras, you have to stop seeing it as any other architectural marvel and start seeing it as a massive, 216-foot-tall geometric proof carved into granite.

Raja Raja Chola I (The visionary emperor), didn't just want a big temple; he wanted a structure that embodied the cosmic order.

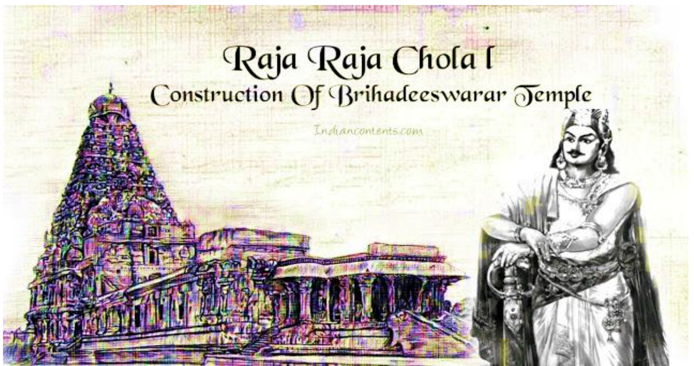


Figure 14: Apex of the Brihadisvara temple

One of the most interesting links of the temple to the Sulba Sutras is the the main tower's (Vimana's)apex. It raises the question: How do you transition a massive square tower into a circular top?
The architects used ratios (In geometric progression) to taper 13 tiers of the tower so that the final platform was the exact size needed to hold the 81.3-tonne granite block.

This wasn't just for aesthetics. By using the Sulba Sutra's circle-square equivalency, they ensured the center of gravity remained perfectly centered. Legend says they built a 6 km long ramp just to get that stone up there—the math had to be perfect before they even started the incline.

A popular local myth is that the shadow of the Brihadisvara's *Vimana* never falls on the ground at noon.
While technically a bit of an exaggeration (it falls on the base/platform), the Sulba Sutra's Gnomon technique is the reason for this precision.

In the 9x9 grid template used for temples like Brihadisvara , the Brahma-sthana (zero-point) occupies the central square. It is considered the "heart" or "navel" of the structure.
In modern terminology, it functions as the geometric origin—the central axis from which the temple's entire Cartesian architectural plane emerges.

By marking the intersection of the diagonals of a perfect square(whose value is $\sqrt{2}$ times the square's side) , architects located this precise mathematical center to ensure the entire building's weight was balanced around a singular, sacred void.
As a result, the temple is so perfectly aligned to the Earth's axis that the shadow stays tight to the structure's footprint

Figure 15:Side view of the Brihadisvara temple

11) Application of the Sulba Sutras in the Modern-era

The Sulba Sutras are also used to date in Hindu rituals for constructing fire altars. In the Vedic tradition, the construction of fire altars (Agni) was not merely a task of masonry but a profound mathematical and symbolic exercise. The Sulba Sutras acted as the engineering manuals for these structures, ensuring that the physical dimensions of the altar mirrored the cosmic order.

You would be mistaken to believe that the application of The Sulba Sutras was just constrained to ancient architecture. While the Sulba Sutras (800–200 BCE) originated as manuals for constructing Vedic fire altars, their geometric and algebraic principles are foundational to several modern technological and educational fields

Using these “ Rules of the cord”, field engineers perform “on-the side verification” of digital designs. For example, if a computer- generated plan for a circular stadium needs to occupy the exact area as the previous rectangular warehouse, transformations as prescribed in the sutras provide a fail-safe method to double check the math.

Figures 16 & 17: Ritual altars

For example, the step-by-step procedural nature of the Sulba Sutras is recognized as an early form of algorithmic thinking, which is a core principle in modern computer science.

Figure 18: Ritual using mud Altar

Also, Some researchers link ancient Indian trigonometry and algebraic approximations to the calculations needed for Celestial Mechanics and space navigation in future lunar or Mars missions.

12) Conclusion:

The approximation of √2 and π, accurate to four decimal places, highlights a deep understanding of number and structure.
What makes this approach truly remarkable is not just the result, but the method: transforming ropes, shapes, and constructions into powerful mathematical ideas.

These ancient texts bridge the gap between ritual and mathematics- showing that mathematics has been both practical and profound.

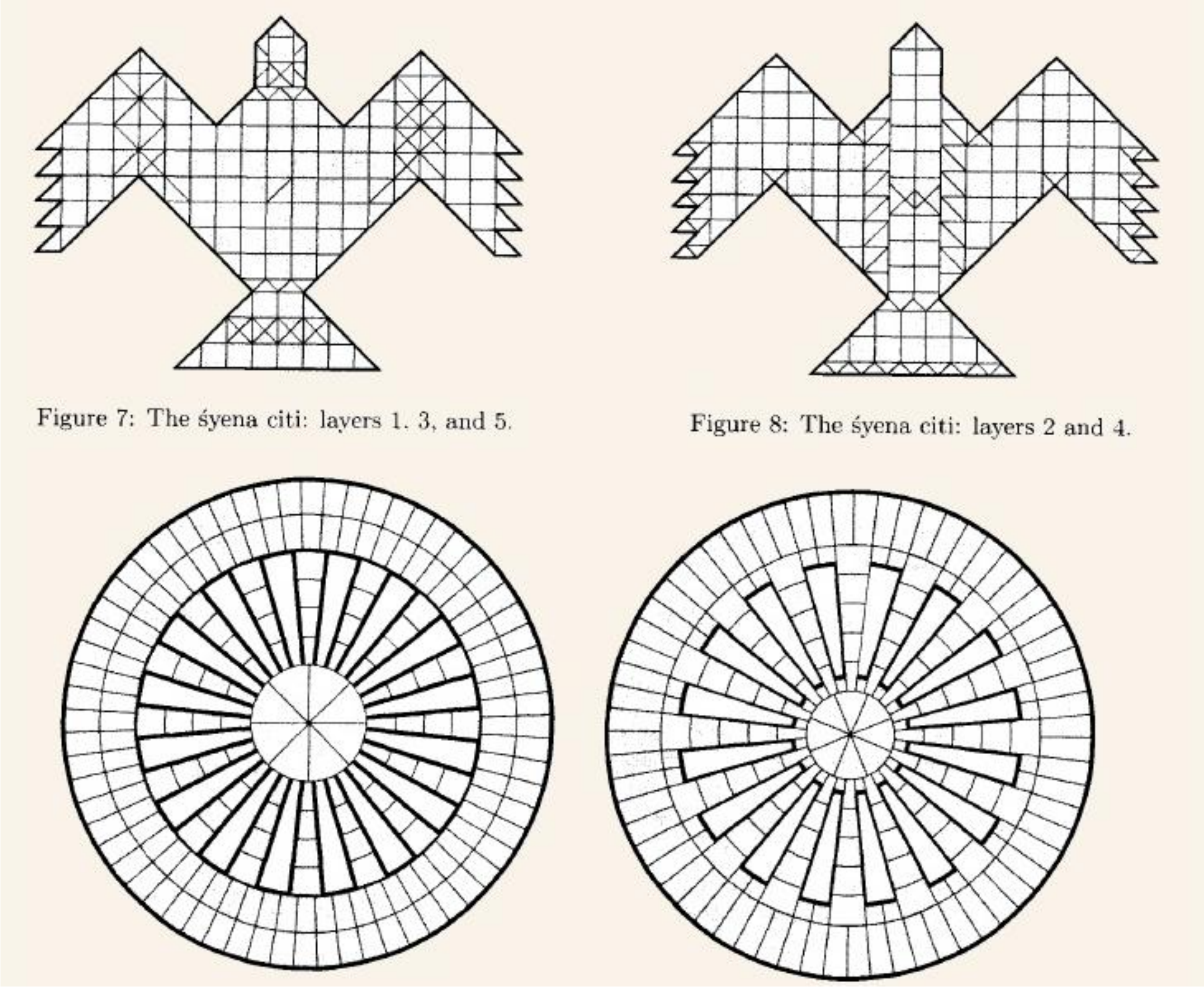


Figure 19 :Śyena Citi (Top two): A fire altar in the shape of a falcon, believed to carry the soul of the sacrificer to heaven.

:Rathacakra Citi (Bottom Circles): A chariot-wheel shaped altar, symbolizing movement and the cycle of time

And the most surprising thing is that centuries before formal mathematics, these ideas were already being explored—not on paper, but with rope, space, and thought

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