The Symbolism behind Mathematical Symbols

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Often when solving mathematics, we are so busy focusing on solving problems that we overlook the most fundamental units of the problems themselves, the symbols. This paper is going to *attempt* to explain the importance and then invent new mathematical symbols. That's correct. The scope of this paper is quite broad given the word limit (*not to mention, the limited mathematical capabilities of the author)*, but it is important to note that many symbols in history of mathematics came to existence merely for ease, and by learning from earlier mathematicians themselves. As of now, the one advantage that we have over earlier mathematicians is the broad library of symbols that already exist.

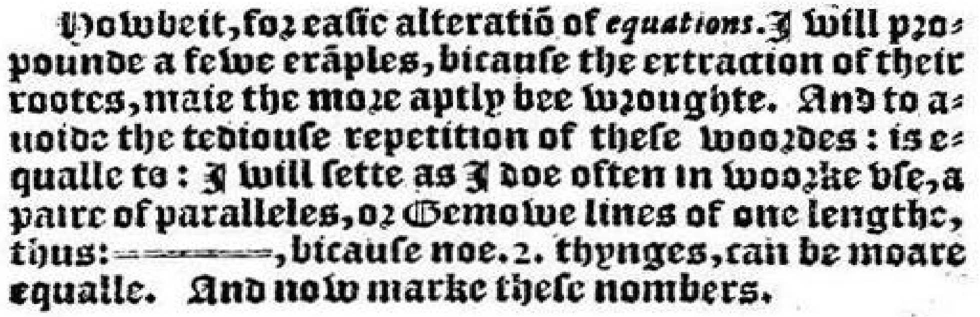
Before doing our initial attempt, we should explore some already existing symbols to get a hint of how these symbols came to be.

The Equality Symbol (=)

Perhaps the most fundamental building unit of an equation in Mathematics, to no one's surprise, also making it the most frequently used symbol in mathematics is the equal sign with about 94% of mathematical expressions containing it.
Prior to the invention of the equal sign, many early mathematicians used to explicitly mention in words that the equation "was equal to" , or other words such as *aequales, esgale, faciunt,* or even some symbols *"æ" and "œ".* (Rojas, R. (2025). *The Language of Mathematics*.)

Early records state that Welsh Physician and Mathematician, Robert Recorde is known to have invented the equal sign by the first use case of the sign appearing in his publication titled *The Whetstone of Witte,* the second part of his arithmetic publication dated 1557.
He states in his book: "*And to avoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, of Gemowe (or twin) lines of one lengthe, thus: = = = = = =, bicause noe 2 thynges can be moare equalle.*"

“And to avoid the tedious repetition of these words: "is equal to" I will set as I do often in work use, a pair of parallels, or duplicate lines of one [the same] length, thus: ======, because no 2 things can be more equal.”


Howbeit, for easie alteratiō of *equations*. I will pro-
pounde a fewe exāples, bicause the extraction of their
rootes, maie the more aptly bee wroughte. And to a-
uoide the tediouse repetition of these woordes : is e-
qualle to : I will sette as I doe often in woorke vse, a
paire of paralleles, or Gemowe lines of one lengthe,
thus: =======, bicause noe. 2. thynges, can be moare
equalle. And now marke these nombers.

By Robert Recorde - http://blog.plover.com/math/recorde.html - The image is an excerpt from the Recorde's book The Whetstone of Witte., Public Domain, https://commons.wikimedia.org/w/index.php?curid=4941779

Simply put, the existence of the equal sign was more or less proposed “to avoid tedious repetition of the words , “ is equal to”, which about *sums* it up. A simple and elegant invention that has saved us from rewriting.
Imagine having to solve a complex question where you have to keep mentioning is equal to several times in each step!

Another fundamental thing to notice about this symbol is the symmetry, a set of parallel lines.

Symmetry in symbols

Most symbols in mathematics appreciate the beauty of symmetry and a difference in orientation simply, I hadn’t noticed this prior to seeing them all together this way, which has really blown my mind *(and given me more hope in discovering my own)*.
Consider some examples:
U, used in Set Theory to express
$\cup$: union of sets,
$\supset$: inclusion (also used to denote logical inference),
$\cap$: intersection of sets, and
$\subset$: inclusion.
The letter U in different orientations represents various operations in set theory.

Similarly, the letter T, is used to express
A^T: transposed vector or matrix (as superscript),
$\vdash$: logical assertion,
$\perp$: contradiction,
$\dashv$: left adjoint.

V, used in logic and algebra to express
$\vee$: disjunction,
$>$: greater than,
$\wedge$: conjunction, and
$<$: less than.
(Rojas, R. (2025). *The Language of Mathematics*. Princeton University Press, Chapter 2, Symmetry of symbols)

This along with the fact that some predicate logic symbols such as the symbol for "There Exists", the Existential Quantifier which is simply a mirrored image of the letter E ($\exists$) and similarly the symbol for "For All" , the Universal Quantifier invented *quite recently* in comparison to other symbols, in 1935, was in similar vein to the invention of ($\exists$), denoted by an upside A ($\forall$).

Many symbols also come to exist from modified Latin, Greek, etc and not just in the context of Mathematics, but in that of other sciences such as Physics, Chemistry, and many more.
So,with a vast library of symbols and characters, and some that we may leave to our own imagination, does this give us a cue to just rotate all the alphabetical symbols in different orientations to achieve new symbols?
Well, not really. You see, simply rotating the symbols will not be sufficient, we also need to do the hard part, that is bring in some rules that will make sense in context of those very symbols. Think of it like making a new word, I can make one right now if I smash random characters, *fjskdfjskjdf* , but does it have any meaning?
So after careful analysis, I have come up with the following symbols to propose in mathematics.

1. The symbol Z (Use case: Distance between two points)

Firstly, this may not be very creative, as there is already the symbol Z in existence, denoting the set of Integers. And secondly, we have other ways to represent distance between two points, most commonly as follows:

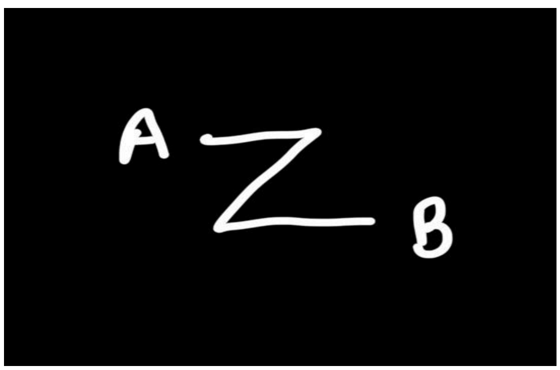
(1,4) and (4,6) or simply using a line between them: A — B.

But this proposal comes in similar fashion to the definite integral symbol, where the lower and upper limits will be written for example, when saying an integral goes from 4 to 8 we could write it as

$$\int_{4}^{8} f(x)dx$$

Similarly, our Z symbol is an elegant way to represent the distance between two points, let's say A and B can be represented as

Figure a : General Symbol representation



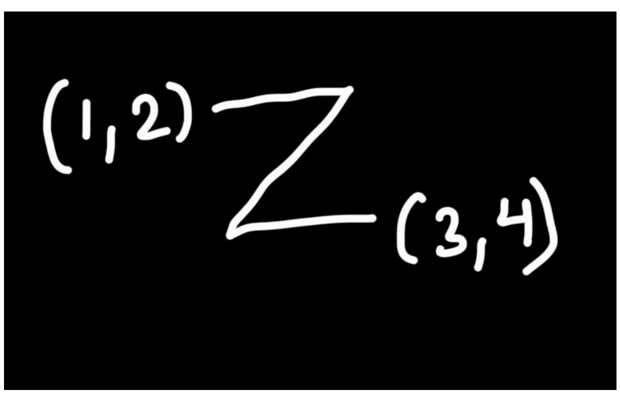


Figure b: An example use case of the points of distance between two points A and B

The definition of the distance would remain the same, for example the above written figure (b) would be calculated as:

A=(1,2) B=(3,4)

$$AB = \sqrt{(3 - 1)^2 + (4 - 2)^2}$$

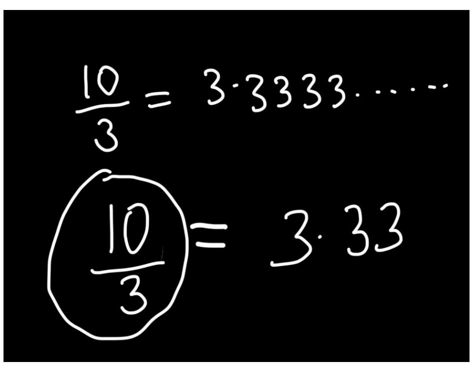
$$AB = \sqrt{2^2 + 2^2}$$

$$AB = \sqrt{8} = 2\sqrt{2} \text{ units}$$

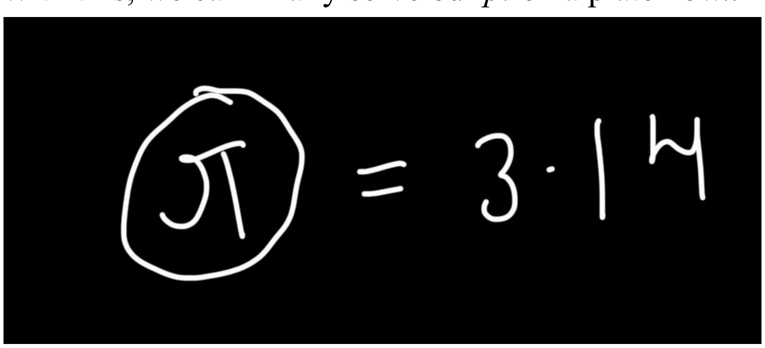
2. Rounding Numbers (literally)

When working with decimal numbers, we often want to round off to the nearest significant integer, based off of the decimal places. But notice how we don't have a symbol for saying this, we merely just say round off or write in the next step using approximation (~). We have symbols for the ceiling function (to round to nearest highest integer) or floor function (nearest lowest integer) but not an approximation or closest significant rounded answer as we might often require. A simple solution could be just enclosing the answer in a circle, quite efficient and simple to get the point of *rounding off* across.

Here is how it'd look:



With this, we can finally serve our *pi* on a plate now..



Quite the *revolution*.

However, there could be drawbacks to our rounding off circle, it is quite clear as the function would get bigger, ironically, our rounding circle would be huge! It would look quite messy and not at all elegant on paper, which is why this wouldn't be a good use case, moreover, this would quickly cause teachers to be confused when their students are already grading themselves. (Marks are usually circled when grading students on a paper).
Quick fixes include using a function definition such as f(x) or x before proceeding to round off large values and for students, as long as they don't use red ink, it should work fine.
For example, x=3.14 and then round off x to get a smaller,cleaner and overall readable representation.

3. Two Pie in my Eye

This proposal comes from a deep place, I had first written this as a journal entry back in 2025, merely because it intrigued me. I wanted to "invent" a unique number that won't just sound funny, but would actually have properties that make it unique. So then came *Two Pie in my Eye*, the number :

$$2\pi i$$

Notice that this number has a unique whole number , 2 which is the only even prime. An irrational number π, and finally an imaginary number i, all this chaos creates a beautiful number. In fact, when you include another chaotic number, *e* and raise the aforementioned quantity, you get

$$e^{2\pi i} = 1$$

(*This equation was not invented by me, I only came up with two pie in my eye.)*

Now that, is awesome. But how do we obtain this result? This number comes from a well known equation is mathematics, by the name of **Euler's Identity**, sometimes referred to as the *most beautiful equation in mathematics*

$$e^{\pi i} + 1 = 0$$

The proof of this equation is fairly straightforward when we use the Euler's Formula,

$$e^{ix} = cos(x) + i\,sin(x)$$

It's crucial to note that the angle x, which should be in **radians**, describes by what *degree* we rotate on the complex plane. This has significant applications in various fields such as in circuits, waves, mathematics, etc.

Plugging in the angle as $x = 2\pi$,

$$e^{2\pi i} = cos(2\pi) + i\, sin(2\pi)$$

We are aware from trigonometry that,

$$cos(2\pi) = 1$$

$$sin(2\pi) = 0$$

Therefore, we simply have,

$$e^{2\pi i} = 1$$

And that proves the result using our infamous, two pie in my eye.

In fact, we can take it a step further and have multiples of two such as *two pie in my eye, four pie in my eye,* etc. all substituted in place of x in e^{x} to yield the same result. But, it is not true for πi. Because that would result in only a half rotation, visually on a complex plane. And

$$e^{\pi i} = -1$$

So we can never say pie in my eye, that's negative and only half true (half rotation). Makes sense intuitively too, when you think you got two eyes, that's why two pie in my eye.

One more interesting part about this equation and the number two pie in my eye is the fact that you come back full circle to where you started. This is why it is a full rotation. This applies well to the field of physics too when looking at periodic motion and oscillations,

$$e^{i\omega t}$$

When $\omega t = 2\pi$, we get

$e^{2\pi i}$, which again denotes one complete oscillation.

These properties enable two pie in my eye to be quite a powerful number. Here, in the context of Euler's formula. There could be much more to explore about this number, but for now, this is a sufficient enough proof of this number being very powerful. So the next time someone asks you "What's your favourite number?" You could just say 6 or 7 and be boring, or true warriors say *two pie in my eye*. It would surely leave room for a longer conversation, a rather interesting one at that.

References:

- *https://commons.wikimedia.org/w/index.php?curid=4941779*
- Equals sign - Wikipedia
- Rojas, R. (2025). *The Language of Mathematics*. Princeton University Press. *(Note: This book is truly a gem for anyone interested about history of maths and symbols as a whole)*