The Twin Prime Conjecture

- **History**

  To understand this conjecture, firstly we must understand the basics. Proposed in 1849 by Alphonse de Polignac, as a special case from a bigger conjecture, which states:
  For every natural number, say k, there are infinitely many primes p such that $p + 2k$ is also prime. The special case of $k = 1$ is the Twin Prime Conjecture.
  This statement is very frustrating for a normal person as it is so simple enough to state that a 11 year-old can easily understand but professional mathematicians debate to this day!

- **The Work**

  Viggo Brun, a mathematician from the 20th century, showed the that the sum of reciprocals of twin prime pairs, as shown

  $$\left(\frac{1}{3}+\frac{1}{5}\right)+\left(\frac{1}{5}+\frac{1}{7}\right)+\left(\frac{1}{11}+\frac{1}{13}\right)+\cdots$$

  converges to a single number. That number is Brun's constant, and it is approximated at 1.902. After nearly a century, in 2013, Yitang Zhang proved that there exists infinitely many pairs of prime numbers for an integer, N, less than 70 million, where there are infinitely many pairs of prime numbers that differ by N.

- **Fun Facts!**

  1. 3,5,7 is the only possible triple twin prime pair.

2. The sum of the numbers in a twin prime pair is divisible by 12, except 3 and 5. This is because of the fact that twin prime pairs are of the form $(6n-1, 6n+1)$, where n is a natural number.
3. The largest known twin prime pair is $2996863034895 \times 2^{1290000} \pm 1$, with each number having 388,342 digits.
4. The multiple of two numbers in a twin prime pair is always 1 less than a square.
5. The fact that the sum of the reciprocals of twin primes is proven to be convergent, but the exact value is not known!

- **My attempt at proving the conjecture**

  Note that this proof may have errors!

  Let us start the proof by defining an 'algorithm'. Say we have:

  $$i, j$$

  in which i and j are both primes part of a twin prime pair and $i < j$.

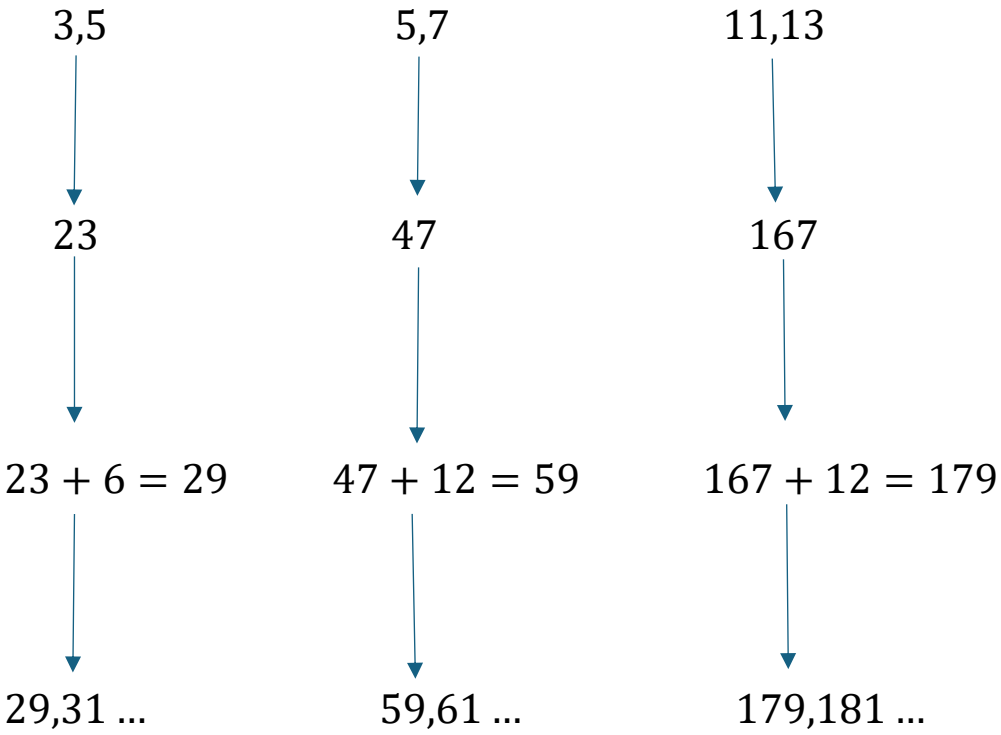
  Now, we will square the largest prime number in the pair, say j. Then, we will subtract 2, which will look like this:

  $$i, j \downarrow j^2 - 2$$

  Also, for the next step, we will at an even number to our previous result such that the smallest possible even number is added to our previous result and get the next smallest twin prime pair, say a, b,

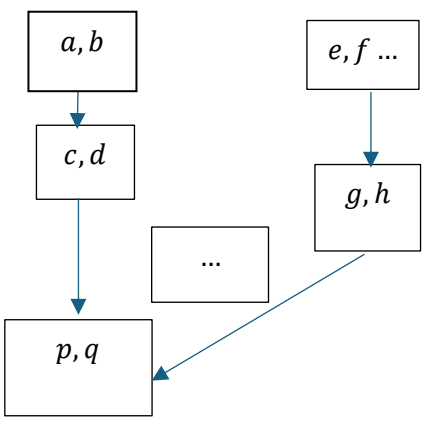
  $$j^2 - 2 + 2n \downarrow a, b$$

where $a = j^2 - 2 + 2n$, where $2n$ is the smallest even number difference to get to a and $a < b$. This process continues indefinitely, without stop. These twin prime chains for different twin prime pairs are as listed. Also, note that there are different twin rime chains as well:



The reason why when we square the number and take away two and add a difference greater than two is because when we add two, the -2 and $+2$ cancel, leaving just j^2, which is clearly not prime. Now, proving the Twin Prime conjecture now just means either of the following cases:

1. The 'chains' meets at one point[s], which means that all of the different 'chains' end at one twin prime, say the 'boss twin prime'. For a quick visualization, here is it will look at if it did meet at a 'boss twin prime':



2. The ‘chains’ end at different points.

Also, note that some of the number chains generated by twin primes are also the same because as we go down the first twin prime chain, say $3,5$, after applying our algorithm, the twin prime is $29,31$. So, if we started from $29,31$, we would get the same twin prime pairs such as $3,5$. Now, let us discuss the points in more detail!

1. For this point, let us assume that the ‘second’ smallest twin prime, say h, i. Now, this is the second smallest twin prime pair, thus when we add an even difference to $j^2 - 2$, it must follow that h, i is closer to $j^2 - 2$ than the largest twin prime, or the ‘boss twin prime’. Now, our third smallest twin prime pair also follows the same logic, so our fourth, fifth and sixth and so on. This means that there is only one possible twin prime chain in this case, which is clearly impossible as we seen from our examples.
2. This is also impossible, as other twin prime pairs can also reach one ‘boss twin prime’ as per our algorithm, the ‘boss twin prime’ is defined as the largest such that $j^2 - 2 + 2n$, so we can add the smallest even difference from the previous result and eventually, all the chains will meet at the ‘boss twin prime’. But from our analysis of our first case, we can clearly see that the ‘boss twin prime’ does not exist!

Therefore, we have ‘disproven’ that two of the cases are possible, thus proving the twin prime conjecture correct, also, again noting that this proof may have errors!

- ## Conclusion

Before I end this essay on the wonderful topic of twin primes, I also want to mention the hard work put into this puzzle by other mathematicians, and there are so many of them that it takes whole new essay on them!

Finally, a quote from Einstein: ‘’Pure mathematics, is, in its way, the poetry of logical ideas’’.