The Optimal Choice For an Individual

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Abstract

What is the best choice for an individual that reduces the competition and maximizes the gain in situations where the choices seem to have the same value to an individual?This starts with a seemingly unimportant question about how an individual crow shall decide which group of bones is the best for the option for it to take,but this also applies to humans, other animals, and any thing that can make a choice.

Contents

Chapter 1

What Should the Crow do?

1.1 The setup

We have a total of 10 crows each independent to make decision(i.e., they decide what they want to do without discussing with the others), and we shall also give them the ability to think and reason like humans. There are two sets of bones from which the crow can choose from,let's call one group 1 which has 10 small bones and the other group 2 has two big bones, which are equally tempting to the crow as the 10 small bones combined as shown below:

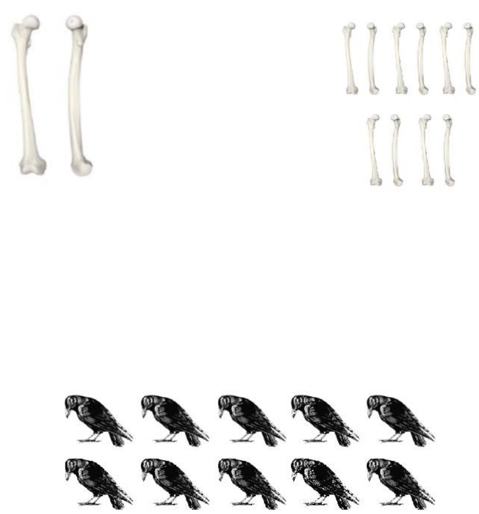

Figure 1.1: The setup

So, from which group shall the crow take and how many shall the crow attempt to maximize his gain and minimize the competition.

1.2 Quantifying the Competition

Let's say an individual assigns some value to each of the items in each group. We shall assign all the bones in group 1 as a value of 1 each and all the bones in group 2 as 5 each giving the total value for each of the groups as 10. Let's give a variable to represent the value of each item call it V. Each group has a different V, V_1 and V_2. We shall also define a few more variables, the amount of competition shall be C, number of competitors shall be n_c, number of items shall be n_i. with these we frame the proportionalities:

$$C \propto n_c$$

more competitors more competition and

$$C \propto \frac{1}{n_i}$$

more the items less competition These make intuitive sense so I wont justify them further.

The value of what the of the items also plays an important role in the competition as the more valuable the item is the more the competition there will be for it.

$$C \propto V$$

The amount of items one wants to attempt also matters as the more they attempt the more they compete but the more they can gain, call this variable T. For our purposes lets say you get as many as you attempt after competing for them.

With all this information we can define what C is. As $C \propto \frac{n_c}{n_i}VT$ we can define C to have a proportionality constant to be 1 as it will make our analysis easier. So,

$$C = \frac{n_c}{n_i}VT$$

Now we have redefined our question, which choice will give the lowest C for the highest value of VT.

1.3 Finding the most probable value for n_c

We have no control over what n_c,n_i and V are but we know what it(usually) is such that we can decide compute C.

let's assume a crow can go to any group and the probability of going to any group is equal.So the probability to go to any one group is $1/n_g$, where n_g is the number of groups. So, the probability to find any n_c number of crows out of N crows(N crows is the Total number of crows minus 1 as we don't want to include the crow evaluating) can be represented by a binomial distribution as we have two options for an individual crow either the crow is there or it is not. So,

$$P(n_c \text{ of } N) = \tbinom{N}{n_c} p^{n_c}(1-p)^{N-n_c}$$

($\binom{N}{n_c}$ read as "N choose n_c" and computed by $\frac{N!}{n_c!(N-n_c)!}$ gives the number of combinations of N items in groups of size n_c, we are using this as we need to add the probabilities of all the possible events which have n_c crows)

p represent the probability of an individual crow going there. Which is $\frac{1}{n_g}$ and $1-p$ gives the probability of that individual crow not going there.

Now what we want to find is which n_c has the highest probability. As we have described our probability of finding n_c crows as a binomial distribution what we want to find is the mode(the value which has the highest probability). Let's say $n_c = x$ and also define our probability of finding x crows as a function of x

$$f(x) = \binom{N}{x} p^x (1-p)^{N-x}$$

We know what N and p are known.

1. when $p = 0$:

   Since $p = 0$ it means $f(x) = 0$ for all values except at $x = 0$, as $f(0) = \binom{N}{0}0^0(1-0)^{N-0}$ which is $1 \times 1 \times 1$ which is 1. So if $p = 0$ then the mode is $x = 0$

2. when $p = 1$:

   since $p = 1$, $f(x) = \binom{N}{x}1^x(1-1)^{N-x}$ is zero for all values not equal to N, as at $x = N$, $f(N) = \binom{N}{N}1^N(1-1)^{N-N}$ is 1 and at any other value the $(1-1)^{N-x}$ will be zero making the whole function zero.

3. when $0 < p < 1$:

What we want is at what value of x is $f(x)$ maximum. So if we compare $f(x+1)$ with f(x) then we can see at what values does the function increase, decrease or remain the same.

$$\frac{f(x+1)}{f(x)} = \frac{\binom{N}{x+1}p^{x+1}(1-p)^{N-(x+1)}}{\binom{N}{x}p^{x}(1-p)^{N-x}}$$

which become

$$\frac{f(x+1)}{f(x)} = \frac{x!(N-x)!}{(x+1)!(N-x-1)!} \times \frac{p}{1-p}$$

after simplifying the factorials

$$\frac{f(x+1)}{f(x)} = \frac{(N-x)p}{(1-p)(x+1)}$$

(a) when the function is increasing:
that is $\frac{f(x+1)}{f(x)} > 1$ then $\frac{(N-x)p}{(1-p)(x+1)} > 1$, from which we deduce $x < (N+1)p - 1$, so the function increases for $x < (N+1)p - 1$

(b) when the function is equal:
that is $\frac{f(x+1)}{f(x)} = 1$ then $\frac{(N-x)p}{(1-p)(x+1)} = 1$,from this we get $x = (N+1)p - 1$, so when $x = (N+1)p - 1$ then $\frac{f(x+1)}{f(x)} = 1$.

(c) when the function is decreasing:
that is when$\frac{f(x+1)}{f(x)} < 1$ so, similarly we get $x > (N+1)p-1$ which means when $x > (N+1)p-1$ then $\frac{f(x+1)}{f(x)} < 1$.

The mode occurs when $f(x)$ is maximum that is $f(x+1) \leq f(x)$ and $f(x-1) \leq f(x)$. Since $\frac{f(x+1)}{f(x)} > 1$ for $x < (N+1)p - 1$ and $\frac{f(x+1)}{f(x)} < 1$ for $x > (N+1)p - 1$, the probability increases till $x = (N+1)p - 1$ and then starts to decrease.

Now finally we shall determine the mode

let $m = (N+1)p$

(a) case 1:when $(N+1)p - 1$ is an integer
if $(N+1)p - 1$ is an integer then m is also an integer.
if $x = (N+1)p - 1 = m - 1$ then $\frac{f(x+1)}{f(x)} = \frac{f(m)}{f(m-1)} = 1$ which means that at $x = (N+1)p - 1$, $f(m) = f(m-1)$ as we saw earlier $f(x)$ was maximum at $x = (N+1)p - 1$ which is at $x = m - 1$ but we saw earlier that $f(m) = f(m-1)$ so that means $x = m$ is also the mode.
So, when $(N+1)p - 1$ is an integer then the modes are $x = (N+1)p - 1$ and $x = (N+1)p$

(b) case 2: when $(N+1)p - 1$ is not an integer
if $(N+1)p - 1$ is not an integer then m is also not an integer.
Let's take $k = \lfloor (N+1)p - 1 \rfloor + 1 = \lfloor (N+1)p \rfloor$, since $(N+1)p - 1 < k < (N+1)p$, we evaluate at $x = k - 1$ and $x = k$(note: $(N+1)P$ is not an integer)
for $x = k - 1$, $x < (N+1)p - 1$ (as $(N+1)p$ is not an integer k will be less then $(N+1)p$), so $f(k) > f(k-1)$ which means the function is increasing up to k.
for $x = k$, $x > (N+1)p - 1$, so $f(k) > f(k+1)$ which means the function is either equal or decreasing after k. So, we can see that the function is increasing up to k and then decreasing. So the mode is at $x = \lfloor (N+1)p \rfloor$.

So finally The mode is defined for all values of p there can be. So now:

$$mode = \begin{cases} 0, \text{ if } p = 0 \\ 1, \text{ if } p = 1 \\ Np + p - 1 \text{ and } Np + p, \text{ if } Np + p - 1 \in Z \text{ and } 0 < p < 1 \\ \lfloor Np + p \rfloor, \text{ if Np+p-1 } \notin Z \end{cases}$$

Now we are almost done ,as we have found the mode we can now find n_c for $p = 1/n_g$ which is :

$$n_c = \begin{cases} 0, \text{ if } p = 0 \\ 1, \text{ if } p = 1 \\ Np + p - 1 \text{ and } Np + p, \text{ if } Np + p - 1 \in Z \text{ and } 0 < p < 1 \\ \lfloor Np + p \rfloor, \text{ if Np+p-1 } \notin Z \end{cases}$$

I shall also clarify that this does not include the crow evaluating as n_c gives the number of of crow that may be there.

1.4 Final Assembly

We place our value of n_c into out equation of C and then we just evaluate.

$$C = \frac{n_c}{n_i} VT$$

If we keep the values of VT to be 1 and then compare which one has the lowest value of C.

$$min(C_1, C_2, ..., C_{n_g}) = C_{min}$$

we set $VT = 1$ for all groups. By making VT to 1, we ensured a fair comparison of competition across groups. The group with the minimum C under this condition has the least competition. Since VT will be our scaling factor, any other group with the same VT value will have higher competition, as their C values are greater than the minimum C found when $VT = 1$. Thus, selecting the group with the lowest C and maximizing VT for the Competition needed.

1.5 Solution

$N = 10 - 1$(as there are a total of 10 crows)

$n_g = 2$

$p = \frac{1}{2}$

So,n_c is:

$$n_c = \begin{cases} 0, \text{ if } p = 0 \\ 1, \text{ if } p = 1 \\ Np + p - 1 \text{ and } Np + p, \text{ if } Np + p - 1 \in Z \text{ and } 0 < p < 1 \\ \lfloor Np + p \rfloor, \text{ if Np+p-1 } \notin Z \end{cases}$$

As Np+p-1 is an integer $\frac{9}{2} + \frac{1}{2} - 1 = 4$ we use the $n_c = Np+p$ as two cases are possible for if $Np+p-1 \in Z$ and $0 < p < 1$ we choose the worst case that is the one with the highest. So, $n_c = 5$.

For group 1 $C_1 = (n_c/n_{i1})VT$, n_{i1} is 10, $V = 1$,T_{max}=10 let's see what is the value for C_1 when VT = 1

$$C_1 = \frac{n_c}{n_{i1}} = \frac{5}{10} = 1/2$$

For group 2 $C_2 = (n_c/n_{i2})VT$, n_{i2} is 2, $V = 5$, T_{max}=2 let's see what is the value for C_1 when VT = 1

$$C_1 = \frac{n_c}{n_{i2}} = \frac{5}{2} = 2.5$$

We see,$C_1 < C_2$

so by comparing we saw that group 1 has a lower value for competition C_1, so we take that and put the highest value for T and we get.

$$C_{1max} = \frac{1}{2} 10 = 5$$

So it is the best to go to group 1 as it has the lowest competition for the amount of value. This works for any number of groups and VT even if they are not the same for all groups as we just want to know which group gives the most value for the least competition.