This essay is about nothing

On a Monday morning, period one, while most of us were still half asleep, my maths teacher decided to begin our introduction to complex numbers in an unusual way. Instead of diving straight into equations, he took on the role of a Brazilian tribesman who “did not understand English” and began asking us a deceptively simple question:

“What is number?”

At first, this felt like a joke. Surely, after years of studying mathematics, we should be able to answer that. But as we tried to explain—counting, quantities, symbols—it became increasingly clear that something so fundamental was surprisingly difficult to define.

And then came an even stranger question:

“What is zero?”

Zero represents nothing. But if it is nothing, how can it be something we use in mathematics?

This essay explores that paradox. It will show that zero is not just another number, but a concept that sits at the very foundation of mathematics—one that is essential, yet deeply problematic. Just be grateful that this isn’t taking place in your Monday period one lesson, because believe me, those are not the ideal conditions of a philosophical crisis.

We began with the obvious answer: numbers come from counting.

Two apples. Three books. Four students.

It feels convincing. Numbers seem tied to real objects.

But the tribesman interrupted:

“The apples are real. But ‘two’? Can I hold it?”

And that’s when things started to shift.

Mathematically, what we were describing is called **cardinality**—the idea that a number represents how many elements are in a set.

If a set has no elements, we write:

$$|\emptyset| = 0$$

This is our first serious definition of zero: **zero is the number of elements in the empty set.**

But that leads to a strange conclusion: zero is a number that describes nothing at all.

To make this precise, mathematicians don't just *assume* numbers exist—they construct them. The logician Kurt Gödel showed that in any sufficiently complex system, there are statements that are true but cannot be proven within the system itself. This suggests that mathematics is not a perfectly closed system of absolute truth. Instead, it is built on foundations we accept, called axioms.

Using set theory, due to Von Neumann, we define:

$$0 = \emptyset$$

This means zero *is* the empty set.

Then we build everything else from it:

$$\begin{gathered} 1 = \{0\} \\ 2 = \{0,1\} \\ 3 = \{0,1,2\} \end{gathered}$$

In general:

$$n = \{0,1,2,\dots,n-1\}$$

So each number is literally the set of all smaller numbers.

This might seem abstract, but it does something powerful: it shows that all numbers are built from a single starting point—zero.

And zero itself is built from nothing – the set containing no elements. However, we have not truly explained nothingness, rather just replaced it with a definition.

At this point, you might argue—like our tribesman—that zero is just a trick. A convenient label for "nothing".

But mathematics doesn't let us get away with that.

Zero isn't optional. It is **forced** by the structure of arithmetic.

In mathematics, zero is defined as the **additive identity**, meaning:

$$a + 0 = a$$

But what if there were two such numbers?

Let's suppose there are two "zeros": 0 and $0'$, both with this property.

Then:

$$a + 0 = a \quad \text{and} \quad a + 0' = a$$

Now consider:

$$0 + 0'$$

Using the fact that $0$is an identity:

$$0 + 0' = 0'$$

Using the fact that $0'$is an identity:

$$0 + 0' = 0$$

So:

$$0 = 0'$$

This proves something important:

Zero is not just a concept—it is uniquely determined.

There is exactly one number that behaves like "nothing".

Instead of asking what zero *is*, mathematics often focuses on what it *does*.

For example, zero has a remarkable property in multiplication:

$$a \cdot 0 = 0$$

This feels obvious—but it actually follows logically from the rules of arithmetic.

Using the distributive law:

$$a(0 + 0) = a \cdot 0 + a \cdot 0$$

But $0 + 0 = 0$, so:

$$a \cdot 0 = a \cdot 0 + a \cdot 0$$

The only way this can be true is if:

$$a \cdot 0 = 0$$

So multiplying by “nothing” always gives nothing—not because we decided it should, but because the structure of arithmetic forces it.

We can prove that **every natural number is ultimately built from zero**, using a method called **mathematical induction**.

What is induction?

Induction is a way of proving that something is true for *all* natural numbers.

It works like a chain of dominoes:

1. Show the first domino falls (the base case).
2. Show that if any domino falls, the next one must fall.

Then all dominoes fall.

Claim: Every natural number can be constructed from zero using the successor function.

We define the successor of a number n as:

$$S(n) = n \cup \{n\}$$

Proof:

Base case:
We already have:

$$0 = \emptyset$$

So the statement is true for 0.

Inductive step:
Assume a number n exists.

Then its successor is:

$$S(n) = n \cup \{n\}$$

This creates a new number.

So:

- From 0, we get 1
- From 1, we get 2

- From 2, we get 3

And so on, forever.

Conclusion:

By induction, every natural number exists because zero exists.

This is the key moment.

If zero did not exist, no other number could exist.

So zero is not just “nothing”—it is the **foundation of everything** in arithmetic.

Zero also creates boundaries.

For example, division by zero is impossible.

If we suppose:

$$\frac{a}{0} = b$$

Then:

$$a = b \cdot 0$$

But we proved earlier:

$$b \cdot 0 = 0$$

So:

$$a = 0$$

This means only zero could be divided by zero—and even then, we run into a problem.

Because:

$$b \cdot 0 = 0$$

for *any* b, meaning:

$$\frac{0}{0}$$

could equal any number.

This breaks mathematics completely—so division by zero is left undefined.

Again, zero isn't just passive—it defines what is *allowed*.

So $0/0$is not just undefined — it is indeterminate. It breaks the rules completely, showing that zero sits at a boundary where our usual logic stops working, an issue that becomes even more interesting in calculus.

Consider the limit:

$$\lim_{x \to 0} \frac{\sin x}{x}$$

Direct substitution gives $0/0$, which tells us nothing. But by analysing the behaviour of the function, we find:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Here, zero is not a number we plug in or reach, but something we approach.

Calculus resolves the paradox by avoiding zero itself and instead studying what happens infinitely close to it.

Zero also plays a crucial role in defining existence in algebra.

To solve equations like:

$$x^2 - 4 = 0$$

we rewrite them as:

$$x^2 = 4$$

The idea of solving equations depends on identifying when expressions equal zero. In this sense, zero acts as a reference point — a boundary between positive and negative, between solutions and non-solutions.

Without zero, equations lose meaning.

In many ways, mathematics is built on distinguishing something from nothing. Sets either contain elements or they do not. Functions either map to a value or they do not. Statements are either true or false.

Zero sits at the centre of this distinction.

It is both a number and the absence of number. It behaves like something, while representing nothing.

Historically, zero was not always accepted. Ancient Greek mathematics avoided it entirely. Even when it appeared in other cultures, it was treated cautiously. It was only when zero was fully embraced — particularly in Indian mathematics — that algebra could properly develop.

So something that represents nothing turned out to be essential for everything.

Perhaps the strangest part is this: we cannot prove anything exists mathematically without first defining what it means for something to equal zero.

Equations, limits, structures — they all rely on it.

So, mathematics depends on nothing.

And that is the real paradox.

Zero is both the simplest number and the most conceptually difficult. It is the foundation of modern mathematics, yet it represents the absence of everything.

It is not just another number.

It is the boundary between something and nothing — and everything we build in mathematics depends on where we place it.

By now, the tribesman's question feels much harder.

"What is zero?"

We can answer in multiple ways:

- It is the cardinality of the empty set
- It is the additive identity
- It is the foundation of the natural numbers
- It is a boundary in calculus
- It is a structural element in algebra

And yet, none of these answers tell us what zero *is* in a physical sense.

That Monday morning lesson didn't give us a final answer—and maybe that's the point.

But, maths had given us something better: not a definition, but a structure.

Zero may not exist physically. It may not be something you can point to or hold.

But mathematically, it is unavoidable.

It is the point from which numbers grow, the identity that preserves structure, and the boundary that defines entire fields of mathematics.

It is nothing—and yet essential.

It does not exist—and yet everything depends on it.

It is nothing - but that is exactly why it matters.

Zero does not exist in the world.

But without it, no mathematical world could exist at all.

Perhaps the most surprising conclusion is this:

Mathematics, often seen as the most certain of all disciplines, is built upon ideas that we cannot fully justify.

And it all begins with nothing.

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