The Mathematics and Physics Behind Skating.

Introduction

Figure skating throughout the years has captivated the eyes of the viewer through its blend of art and athleticisms, where jumps and turns seem so effortlessly done. The moment the blades touch the ice, choreography and elegance overwhelm the atmosphere, however beneath all that grace, lies a rigorous foundation of mathematics and physics that knit the possibilities of the spectacle we see on our screens.

Friction and Ice

Consideration of how to optimise performance doesn't start when the tricks do, but the moment the blade etches into the ice. This is primarily due to the coefficient of friction between the ice and the figure skating blades. With the standard parameters being the weight of the skater, pressure between the ice and blades, temperature and area of the blade in contact with the ice. The standard equation being:

$\mu = F / N$

μ = coefficient of friction

F= Frictional force

N= normal contact force

In this given context the normal contact force refers to the skater's weight. Hence the equation being rewritten as $\mu = F / W$

The weight of the skater is concentrated into the thin blades of the skates, exerting pressure (Pressure=Force/Area). The pressure slightly lowers the melting point causing the ice to melt, creating a thin layer of water to form acting as a lubricant and reduces friction and μ facilitating smoother glides and movement. This is why skaters regularly sharpen their blades, in order to prevent it from dulling down and increasing the surface area of the contact between the blades and ice, which may increase resistance against the ice. At lower temperatures the ice is hard and provides more resistance. As the temperature increases the ice melts, resulting in a smaller coefficient of friction.

Angular momentum

To rotate in the air, the skater must generate angular momentum at takeoff.This describes the tendency of an object to maintain rotational motion. Angular momentum is a crucial aspect in our understanding of how skaters are able to spin so fast. It allows skaters to control spin speed and complete rotational jumps by conserving rotational motion. It is defined as the product of the skater's moment of inertia (I) and angular velocity (ω).

$L = I \times \omega$

Where moment of inertia refers to a physical quantity that measures an object's resistance to changes in rotational motion about a specific axis and is crucially affected by its mass distribution.

$I = mr^2$

Where m is the mass and r is the distance from the axis.

The further the mass is from the axis of the rotation - the centre of gravity - the greater the moment of inertia, the more difficult it is to execute the spin as well as changing the speed and halting the spin. Double the distance of your arms from your body and the moment of inertia increases by 4. Overall just making it a right pain to seamlessly move through the air. So how do we decrease the inertia? Simply tucking in your arms. And this is where conservation of angular momentum comes in. Angular momentum is conserved as long as no external torque acts upon the system. In the air angular momentum is constant. This means that $I_1\omega_1=I_2\omega_2$. So when the skater tucks in their arms. I decrease and ω must increase to compensate. Thus increasing the speed. Skaters utilise this principle by manipulating their mass distribution via other forms as well such as drawing the legs close to the body to the position of the chin by tucking it in, all which brings their mass closer to the axis. Whilst jumping the skater's center of gravity follows a parabolic path that is composed of its vertical displacement and horizontal displacement.

Patterns and Blades

Geometric patterns were once a crucial and intentional aspect of figure skating demonstrating skill and control that involved skaters creating circular patterns on the ice. One of the most noticeable ones being the figure eight. This is formed by two circles of equal size tangential to each other typically three times the height of the skater. The curvature of the skates greatly aid their performance and how these patterns are formed. The bottoms of ice skating blades are slightly curved, allowing the skater to angle and tilt to form these patterns. The curvature determines how much of the blade touches the ice at any given moment. A bigger circle means more contact with the ice. A tighter circle means less blade contact directly aiding on how quickly the skater can pivot as there is less resistance to rotation. However it also provides less stability and requires more control.

Conclusion

Figure skating is far more a display of artistry but rather a powerful demonstration of how mathematics and physics govern every intricate motion, from the moment the blades touch the ice to when bodies breeze and turn through the air. What appears to be effortless is a result of meticulously controlled laws and forces that skaters have enabled to manipulate their performance to the highest standard. Ultimately I hope to show that physics and maths isn't just a textbook, exam or equations of a whiteboard made to memorise but rather principles that are continuously expressed through artistic performance that have captivated our sights.