Why hexagons are the best shape

A hexagon can be defined as a plane figure with six straight sides and angles. Most of us encounter this shape at around age six or seven - usually on a honeycomb, a football, or a set of bathroom tiles - and think little more of it. This essay argues that was a grave mistake.
The claim I intend to defend is this - **hexagons are the best shape**. Now I appreciate that "best shape" sounds suspiciously like a matter of opinion. That is precisely why I shall not argue it as one. Instead, I will employ a beloved technique from the mathematician's toolkit- *proof by contradiction*.

The method is simple. We assume the opposite of what we want to prove “that hexagons are *not* the best shape” and then watch that assumption collapse under the weight of its own absurdity. By the end, we will have shown that in a surprising variety of real-world problems, choosing any shape other than a hexagon leads to an outcome that is strictly worse. The hexagon, it turns out, has been quietly solving optimisation problems long before mathematicians got around to formalising them.

A note on method: *The following is not a proof in the strict axiomatic sense. It is however, considerably more convincing than simply saying "hexagons look cute," which is, frankly, also true.*

The Structure of Our Proof

Let us state our assumption clearly:

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Assume, for contradiction, that hexagons are NOT the best shape.

Then there must exist some shape S ≠ hexagon that performs optimally
across the problems we consider.

We will show for each problem, S performs strictly worse than the
hexagon, yielding a contradiction. (eventually)
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We will test this assumption across four conveniently chosen problems. Should another shape outperform the hexagon in even one of them, our proof fails and we must accept defeat. Dear reader, the hexagon is not worried.

Problem 1 - The Game Map Problem

Lets say you are an ambitious video game developer tasked with creating the map for a new game. Players must navigate freely, interact with objects, and also move to arbitrary destinations. Your map will be made of tiles, and you must choose the tile shape.
You begin, sensibly, with the simplest polygons.

Trial Shape: The Triangle

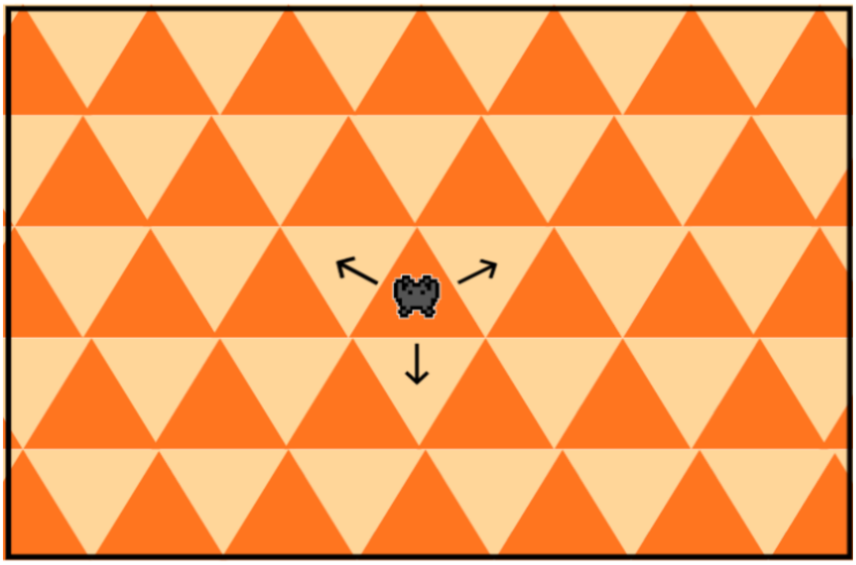

Figure 1

A triangular grid does tile the plane without gaps (*as seen in figure 1*), which is a point in its favour. However, the movement properties are dreadful. Each triangle has only three neighbours, and crucially, they alternate in orientation -some point up, some point down. This means there is no consistent notion of "moving forward": a step in one direction lands you on a differently oriented tile, forcing awkward zig-zag paths. A player trying to walk in a straight line will feel as though they are navigating a sequence of funhouse mirrors.

Trial Shape: The Square

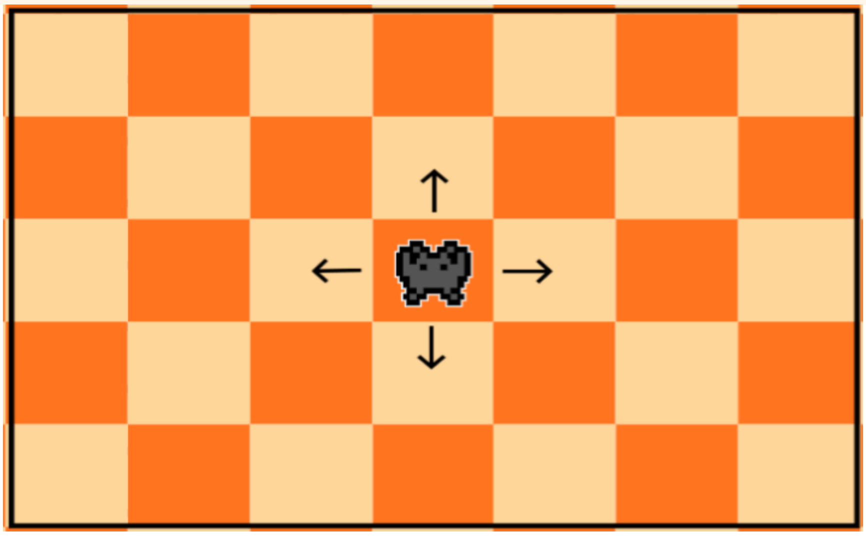

Figure 2

The square is a considerable improvement. It tiles neatly, and movement in the four cardinal directions is intuitive. This is why so many classic games such as chess, early Zelda, Minecraft's top-down modes used square grids. However, the square grid has a well-known flaw: diagonal movement is not equal to cardinal movement. Moving diagonally covers a distance of $\sqrt{2} \approx 1.414$ tile-widths, whereas moving horizontally or vertically covers exactly 1. This inconsistency is not merely aesthetic; it affects combat range, line-of-sight, and any mechanic that depends on distance. Developers either ignore this (introducing unfairness) or correct for it with additional code (introducing headaches).

Trial Shape: The Hexagon

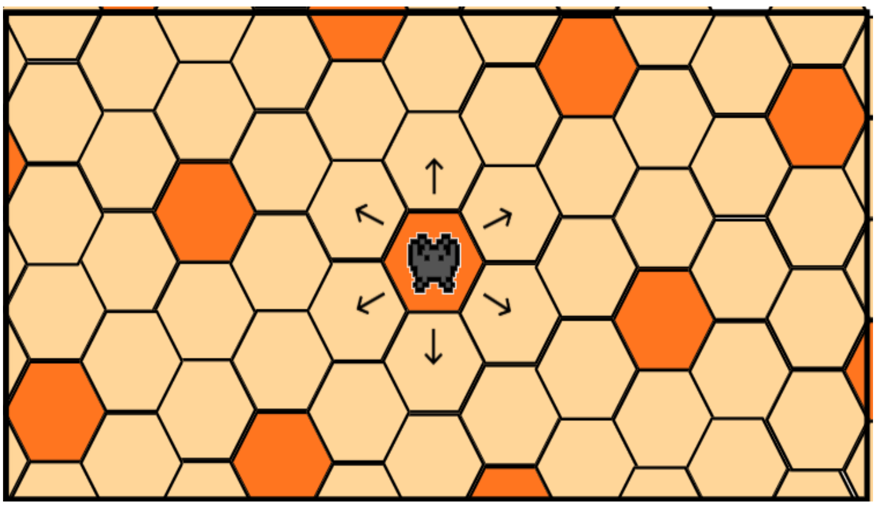

Figure 3

Now consider the regular hexagon. It tiles the plane without gaps, just like the triangle and square. But it has a property that neither of those shapes can claim: every hexagon has exactly six neighbours, and every one of those neighbours is the same distance from the centre. There is no diagonal anomaly. A move in any of the six directions travels precisely the same distance. This makes hexagonal grids natively fair: range-based mechanics, line-of-sight, movement costs - all behave consistently without extra correction.

This is not a coincidence. It is a consequence of the hexagon's internal symmetry. And it is why strategy games from *Civilization* to *Settlers of Catan* have adopted hexagonal grids whenever spatial fairness matters.

Our assumption is already beginning to crack - much like any non-hexagonal structure under stress, as we shall shortly demonstrate.

Problem 2 - The Lazy Bee Challenge

A honeybee faces a genuinely difficult engineering problem. It needs to store as much honey as possible while using as little wax as possible, since producing wax is metabolically expensive. The cells must tile perfectly - no gaps, no wasted space - and the total length of shared walls between cells should be minimised to reduce material use.
This is, mathematically, an isoperimetric problem: among all shapes that tile the plane, which achieves the greatest area per cell for a given total wall length? Our bee, being a sensible engineer, tries a few options.

Trial Shape: The Circle

Circles maximise area for a given perimeter individually - a well-known result from classical geometry. Alas, circles cannot tile the plane. They leave curved gaps at every junction, which bees would need to fill with extra wax or leave as wasteful void. The circle, despite its individual efficiency, is a collective disaster.

Trial Shape: The Triangle or Square

Both tile the plane, but consider what happens to the perimeter-to-area ratio. A square of side length s has area s^2 and perimeter $4s$. A regular hexagon with the same area has a shorter total perimeter. The more sides a regular polygon has, the closer it approaches the circle's efficiency - and among the three polygons that tile the plane (triangle, square, hexagon), the hexagon has the most sides. It therefore achieves the best area-to-perimeter ratio of any tessellating polygon.

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For equal areas:
Perimeter of equilateral triangle: square: regular hexagon
≈ 1.354 : 1.128 : 1.000 (relative to hexagon)

→ The hexagon requires the least wax per unit of honey stored.
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This result is known as the Honeycomb Conjecture, and it was formally proved by mathematician Thomas Hales in 1999. The bees, evidently, did not need to wait for the proof. They had been implementing the optimal solution for millions of years - a fact that should give any triangle-preferring developer pause for thought.

Problem 3 -The Speaker Grille Problem

You are an audio engineer designing the perforated grille that sits over a loudspeaker. You need two things simultaneously: maximum open area (so sound passes through cleanly) and maximum structural rigidity (so the grille does not deform or rattle). These goals are in tension - more holes means less material, which typically means less strength.

Square grid-
Moderate open area; weak at joints and the shear forces concentrate at right-angle corners, causing stress cracking
Random / circular grid-
High open area possible, but uneven distribution creates resonance nodes - some frequencies attenuate, others amplify
Hexagonal grid-
High open area, even distribution and each node has six members meeting at 120°- the most structurally stable angle

The key is the 120° interior angle. When three members meet at 120°, forces distribute evenly across all three - no single member bears disproportionate load. By contrast, the 90° corners in a square grid create localised stress concentrations, which is why square grilles are more prone to warping. This same principle explains why hexagonal mesh is used in everything from speaker grilles to industrial filters to crash-test barriers.

Problem 4 - The Bulletproof Panel Problem

Finally, consider a materials engineer designing a lightweight protective panel (the kind used in body armour, aerospace shielding, or impact-resistant packaging). The panel must absorb and distribute impact energy as widely as possible. A force concentrated in a single point will punch straight through; a force spread across a wide area will not.

A solid flat plate concentrates impact force at the point of contact. Conventional grid structures help somewhat, but the geometry matters enormously. When an impact strikes a hexagonal cell, the cell walls deflect and transfer load to all six neighbours simultaneously. Each of those neighbours then transfers to their six neighbours, and so on - producing a cascade of distributed energy absorption that propagates outward in all directions with remarkable uniformity.

Square cells, by comparison, transfer force primarily along four axes, leaving diagonal paths under-supported. Triangular cells distribute well but require more material for equivalent strength. The hexagon achieves the optimal balance: maximum force distribution with minimum material weight - which is why hexagonal core structures are found inside aircraft panels, bicycle helmets, and advanced body armour.

Incidentally: *the Boeing 787 Dreamliner uses hexagonal honeycomb panels in its fuselage and wings. The hexagon is not merely the best shape for video games and bees. It is also trusted with keeping several hundred people alive at 35,000 feet.*

Conclusion - The Contradiction Is Complete

Let us return to where we began. We assumed, for contradiction, that hexagons are not the best shape. We then considered four independent problems - game map design, biological storage efficiency, acoustic engineering, and impact protection - and in each case found that the hexagon either matched or outperformed every competing shape.

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We assumed: ∃ shape S ≠ hexagon that is "best".
We showed: in Problems 1–4, S performs strictly worse than the
hexagon.
This contradicts our assumption.
Therefore: hexagons ARE the best shape.
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Is this a watertight proof? Strictly speaking, no - we have only considered four problems, and a true proof would require exhaustive consideration of all possible optimisation criteria. But I would argue that four independent domains of human endeavour - game design, evolutionary biology, audio engineering, and aerospace materials science - all converging on the same shape is not a coincidence. It is a pattern. And patterns, in mathematics, have a habit of turning out to be true.

The hexagon did not ask to be optimal. It simply is. The bees worked it out millions of years ago. The engineers caught up eventually. Perhaps it is time the rest of us did too.

Q.E.D. - Quite Evidently, done