Bases: Transdecimal, Subdecimal, and the Numerals themselves.

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1. What is a base exactly?

In this essay, a base will be defined as a numeral system that uses digits to represent values, a base will have n digits because it is base n, starting with zero, and going up, And the notation that the number is in a different base will be notated with a subscript version of the base. So in base 7, the digits are 0, 1, 2, 3, 4, 5, and 6 which is 7 digits in total. And each place value represents a different power of 7, so 635_7 equates to $(6*7^2)+(3*7^1)+(5*7^0)$ (calcualations all in base 10) and can be simplified to $294+21+5=320_{10}$. Now that you probably know what a base is I must mention that the base refers to a base (see above) rather than the base of an exponent. So without further ado, go down there and read the rest of the essay.

2. Decimal? I think we can do better.

Everyone uses the decimal system for maths, right? But have you ever considered that the decimal system is sub-par rather than other number systems? For instance, the dozenal system is much more useful, due to its many factors and easy division rules (for the most part). Many of the things we use are dozenal-centric. Eggs, time, music, and much more. The numerals used for values 10 and 11 can vary from person to person. Personally, I would use the Argam numeral set. (We’ll get to that later.) There’s also the hexadecimal system and binary that computers use in everyday life. There’s also arbitrary bases that nobody uses due to their incapability of simple division rules, and also most of them are prime numbers anyways. There are also bases below 10, negative bases, fractional bases, but then again, the fractional bases are pretty much arbitrary because in base “p”, there are “p” number of digits, but that wouldn't work to have say, 1.5 digits, that’s just not possible. But moving back to integer bases, another commonly used base is senary (base 6) using numerals 0, 1, 2, 3, 4, and 5, which is also convenient because of its relation to dozenal. There are also unusable bases like unary and nullary, which have no convenience because unary has 1 digit (Basically tally marks but worse.) and nullary doesn't really have any digits, moving on from tiny bases, lets move on to the absurdly large bases, there are bases that have no use, and ones that have some uses, like base 64 (very useful) and then there's bases like base 61 which has absolutely no uses.

3. Cool, but how do I express these?

For bases 10 and below, we can use digits from the Hindu-Arabic numeral set to express values. but for bases above ten, is where it gets complicated. There are no defined numerals for transdecimal numeral systems, so there are no defined methods of value expression above base 10, right? Well this is where I introduce the alphanumerics, non-unicode expressible sets, and more. One numeral set I will mention is the Alphanumeric system, using letters that correspond to certain values, this is often used in base 36. To your right is a list of base 36 alphanumerics and their corresponding values.

Alphanumerics in Hexatrigesimal (base 36)

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	A	B
12	C	D	E	F	G	H	I	J	K	L	M	N
24	O	P	Q	R	S	T	U	V	W	X	Y	Z

There are also ancient numeral sets that nobody uses, like the Babylonian sexagesimal (base 60) numerals, that use a decimal-centric numeration method to express values in sexagesimal. And there are also other numerals that people use in dozenal. Most people use “dek” and “ell” representing an “X” like numeral and an upsidedown 3 respectively. or an upsidedown 2 can be used for dek. (Proposed by Sir Issac Pitman.) There are other bases that have any interpretation of their numerals, You can make your own numeral set and use it however you want.

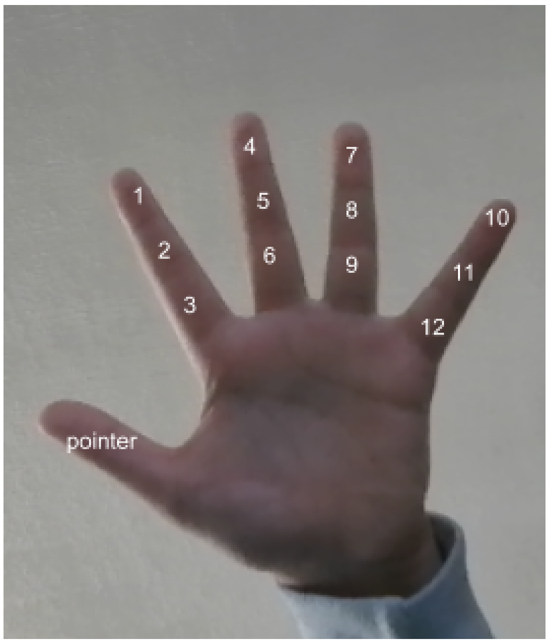
4. Numbers have no end.

As the heading suggests, can we make a base with no definitive 10? Base infinity, which I call aperial has no definitive place value higher than the ones, this is because infinity is merely a concept rather than an actual number, it cannot be visualized, so the idea of having powers of infinity is merely useless. But when we run out of letters, what happens then? Well worry not because in the 1980’s a man named Michael deVlieger created a numeral set called “Argam” which was first made for dozenal, and later was extended into vigesimal (base 20) and then trigesimal (base 30) in 1991, later in the 2000s, Argam was extended further with a multiplicative definition, let’s take digit 27, named “trine”. digit 27 shouldn’t convey the “2 times 10 plus seven-ness”, but rather the three-ness because it is three cubed. This helps convey an non-decimal-centric idea of the mathematical composition of a number. Argam today has 480 numerals, people have tried extending the numeral set, one notable extension is the Valerian extension, which plans to go up to 2520 numerals. Below will be a list of argam from 0-120, with their names (ismaargam) and prime factorizations, please note that I cannot show what the Argam numerals look like as they are not supported in unicode. But they can be found here: [Argam Typeface Extension - Transdecimal Observatory]

+	0	1	2	3	4	5	6	7	8	9	10	11
0	zero	one	two 2	three 3	four 2^2	five 5	six 2×3	seven 7	eight 2^3	nine 3^2	dess 2×5	ell 11
12	zen $2^2\times3$	thise 13	zeff 2×7	trick 3×5	tess 2^4	zote 17	dine 2×3^2	ax 19	score $2^2\times5$	tress 3×7	dell 2×11	flore 23
24	cadex $2^3\times3$	quint 5^2	dithe 2×13	trine 3^3	caven 4×7	neve 29	kinex $2\times3\times5$	sode 31	twive 2^5	trell 3×11	dote 2×17	kineff 5×7
36	exent $2^2\times3^2$	mack 37	dax 2×19	trithe 3×13	kinoct $2^3\times5$	lume 41	exeff $2\times3\times7$	sill 43	cadell $2^2\times11$	kinove $3^2\times5$	diore 2×23	foss 47
48	exoct $2^4\times3$	effent 7^2	kiness 2×5^2	trote 3×17	cadithe $2^2\times13$	sull 53	exove 2×3^3	kinell 5×11	sevoct $2^3\times7$	trax 3×19	deve 2×29	clore 59
60	shock $2^2\times3\times5$	ark 61	diode 2×31	senove $3^2\times7$	twex 2^6	kinithe 5×13	exell $2\times3\times11$	kale 67	cadote $2^2\times17$	triore 3×23	sevess $2\times5\times7$	calse 71
72	octove $2^3\times3^2$	scand 73	dimack 2×37	kinchick 3×5^2	catax $2^2\times19$	sevell 7×11	exithe $2\times3\times13$	tite 79	kintess $2^4\times5$	novent 3^4	dilume 2×41	van 83
84	sezen $2^2\times3\times7$	kinote 5×17	disill 2×43	treve 3×29	octell $2^3\times11$	chrome 89	novess $2\times3^2\times5$	sevithe 7×13	cadore $2^2\times23$	trisode 3×31	doss 2×47	kinax 5×19
96	ozzen $2^5\times3$	mang 97	seneff 2×7^2	novell $3^2\times11$	kent $2^2\times5^2$	ferr 101	exote $2\times3\times17$	cobe 103	octithe $2^3\times13$	setrick $3\times5\times7$	disull 2×53	nick 107
108	cadtrine $2^2\times3^3$	cupe 109	desell $2\times5\times11$	trimack 3×37	setess $2^4\times7$	zinn 113	exax $2\times3\times19$	kinore 5×23	caneve $2^2\times29$	novithe $3^2\times13$	diclore 2×59	sevote 7×17
120	hund $2^3\times3\times5$	Argam Names and Prime Factorization Table 0-120 Fun fact: The names of most of the prime numbers come from elements of the periodic table, Clore derives from chlorine.										

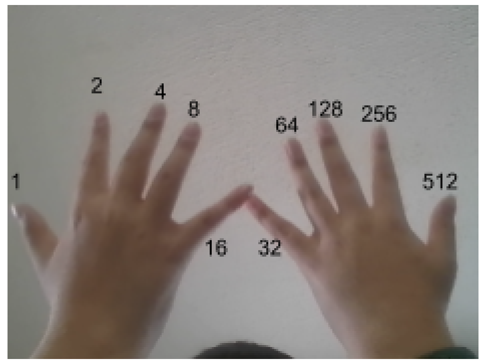
5. Crumple those papers!

OK, let’s go old school for a second, Let’s use our hands. We have 5 digits on each hand, (More or less depending on your hand situation) and we can push this limit. Using a dozenal system, we can count to 144 on our 2 hands. (see the image below.)



debut of my own hand for dozenal counting!

In the image shown, the pointer is used to indicate the number, the other fingers have 12 segments combined so the pointer can move to indicate what number they want to communicate, the other hand is used for the same purpose but it counts dozens. But this is not the limit. We can go further with drumroll please… BINARY! Now on your right is a picture of the binary counting I use, starting from the left thumb to the right is increasing powers of two. If you need to, you can reverse this system to the other side starting from the right thumb and going up to the left. A binary value like 0010110100 has the digits with 1 with open fingers and digits with 0 have closed fingers. With this system, we can count up to 1023. With a senary system, we can get to 35 by using one hand for the ones and the other for the tens. Since we can use zero through five fingers like the senary digits 0 through 5, that makes things easier.



6. [no comment]

Thanks for reading this essay! Now at this point, I’ve ran out of ideas to ramble about, so I’m gonna be brutally honest with you. The inspiration to write this essay was from the Youtube video about this essay competition and a mix of my interest in Argam/miscellaneous numerals systems, and this sounded fun. This whole thing is a manifestation of my interests and my random motivation from “somewhereland”. Now I will just ramble a little more to get more words so just a just stick around for just a little more