

Why Geometry Separates us, but Topology binds us.

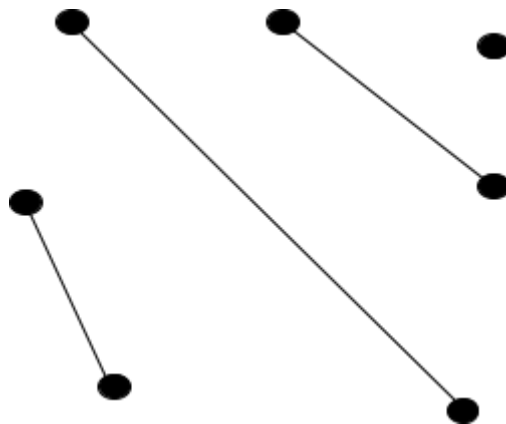
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- Introduction:

Imagine you are on vacation, on a remote island in the middle of the ocean. You meet an elderly fisherman, who happens to have gone to school with your father. This is not coincidence or sheer luck, but in Mathematics, is the very property of a **Social Graph** (a network of people and relationships). The Earth being so physically vast & enormous, is mathematically “collapsing”. We are not just some flesh scattered across the planet, but rather condensed into an interconnected web where everybody lies just a few “handshakes” away from each other.

- The Connection (Nodes & Edges):

We must avoid the complex nature of human emotion , & look at the structure of society itself. In **Graph Theory** , we represent the world as a Graph (G) . Every human on that graph is classified as a Vertex (V) , or a Node ; a simple dot on the diagram. Each and every friendship, relationship & acquaintance being an Edge (E), a line that connects two dots on the Graph.

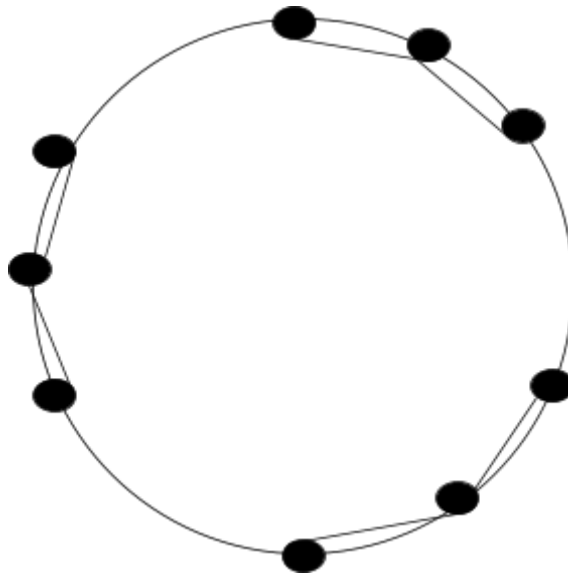


$$G = (V, E)$$

Where G (the diagram), represents the dots as humans scattered across it (V), and the relationships between those humans being the edges (E)

- The Boring World (Regular Lattice Structure):

Let us say I desired to meet the highly esteemed Dr Tom Crawford (it is still a dream), in our boring lattice, the process of me meeting the greatest maths professor of all time would require me knowing people to my left and right **only**.



Each Vertice (V) on the Graph represents a human, with the lines connecting the other dots being the Edges (E)

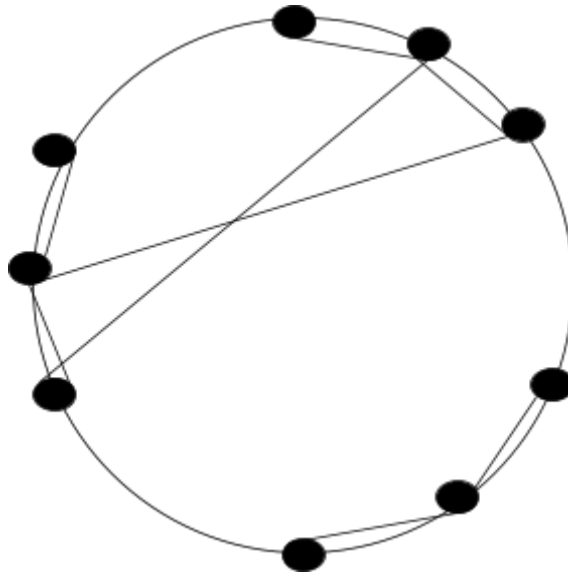
We use the formula:

$$L = N/2$$

Where if the number of people to get to Dr Crawford is 10,000 ($N = 10,000$), the message would go about 5000 people first. If it took the world's population, let us say $N = 8$ Billion people, I would need to be correlated with 4 Billion people first. I'd be long dead before I get to meet Dr Tom Crawford.

- The Small World (Watts - Strogatz Lattice):

In 1998, mathematicians Duncan Watts & Steven Strogatz discovered if you take the “boring world” model, & added a handful of edges across the model, the number of “handshakes” (L) it would take to meet someone would significantly shrink. If it took the same 10,000 people to get to Dr Tom Crawford, though this time I’d coincidentally be related to an undergraduate maths student at Oxford University, the sheer size of the handshakes it would take before ($L = 5000$), would be a single digit now.



These random lines that connect one person with another across the world displayed on the Watts-Strogatz model, they act like wormholes. The connections between these two Vertices on the model are not simply just “connections”, but I’d rather label them as a highway that you can take advantage of to meet thousands of other people across Earth. It uses the same localness vibe from the boring world, though with the addition of a few random connections, it gives us the heeby jeebies of a small world.

- Math of “Six Degrees” ($L \sim \log N$):

Using the Watts-Strogatz model, an equation relating to the model would be:

$$l = \frac{LN(N)}{LN(K)}$$

Where L is the average path length, N is the population, and K is the number of interactions/friendships

Let us assume for the sake of calculations, that $N = 8,000,000,000$, and the average person on planet earth has 100 acquaintances, so $K = 100$.

So the calculation would go something like this:

$$L = \frac{LN(8,000,000,000)}{LN(100)}$$

$$L = \frac{22.803}{4.605}$$

$$L = 4.95$$

With this calculation, mathematically, you and I are roughly 5 “handshakes” away from every human on Earth. This is the “Six Degrees of Separation” proven with a calculation. Is that not the coolest thing ever..

- Phase Transition:

A beautiful concept in Graph theory arises, which is Phase Transition. All the edges between the Nodes & Vertices we talked about earlier, imagine adding them all up individually from person to person. Starting the process of adding up, at first you just get isolated islands of relationships and connections. But throughout the process of adding up, you suddenly form up a “Giant Component”. Bringing it back to the imagination of us meeting that elderly fisherman, where he seemingly had gone to school with my father. That anomaly itself is purely just mathematical certainty, where all of us humans are concentrated into a vast plot of land, and many of these edges are mostly limited by bodies of water. In the modern world, social media overcomes that obstacle where all of us humans could be connected in some sort of way. Unless someone is living in total complete isolation, we are all chained up in this world where

mathematically, someone could meet their soulmate someday. Mathematically, I could be Tom Crawford's neighbour.

- The Erdős Number of a Hug:

Earlier, we defined each edge between humans as an interaction, relationship or acquaintance. If we were to then label these edges as physical hugs, we could potentially come up with a hug number. We humans have been interacting for 2-3 millennia, meaning these edges can span out for generation after generation. You hugged your parents, who hugged their parents, who hugged their parents, & down the line one of them could've hugged the King of England. So mathematically, I could only be a few hugs away from him, but I do not think he'd be into that. Using our earlier model and this analysis, our world feels so small where the diameter of humanity across time could be incredibly tiny for all we know. You are likely 20 or so hugs from someone who walked the Earth 200 years ago.

- Conclusion:

In the current era that we live in, even with a growing population, it may seem that people are losing connection with other humans gradually. Theory tells us that isolation & the feeling of being alone is part of human nature & emotion, though it is not mathematically true. Even with huge gaping landscapes and massive oceans separating billions of humans across the globe, topology still binds all of us humans together, with the potential of me and whoever reading this just a few handshakes away from an interaction or a friendship. Next time if you do ever feel alone, remember that out of 8 billion people on this planet today, the person you may need might just be 3 handshakes close to you.

I hope you enjoyed this walkthrough of the topic. It is truly unfathomable to think that mathematically, you can meet someone across the globe if you have a friend in another country. The math behind this is so incredibly interesting you think to yourself like hm, I wonder if my idol is literally my relative or something... Regardless though, whoever may be reading this, maybe one day might cross paths, knowing that you have just found out through my

explanations in this essay that we probably are really close to meeting one day, but if not I hope you enjoyed this and I am totally sure to not make any more jokes throughout.

- References:

https://en.wikipedia.org/wiki/Six_degrees_of_separation

<http://infolab.stanford.edu/~ullman/mmds/ch10.pdf>

<https://www.cse.chalmers.se/~sattler/docs/diploma-thesis.pdf>

<https://www.sciencedirect.com/topics/mathematics/renyi-random-graph>

<https://snap.stanford.edu/class/cs224w-readings/travers69smallworld.pdf>

https://worrydream.com/refs/Watts_1998_-_Collective_dynamics_of_'small-world'_networks.pdf

<https://snap.stanford.edu/class/cs224w-readings/granovetter73weakties.pdf>