Why an Orchestra Can Never Be Perfectly in Tune

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1. Before the Music Begins

Before an orchestra becomes music, it first becomes listening. The hall is still, the conductor waits, and I lift my oboe to play the note that everyone else will build around: A. For a few seconds, the sound seems to gather the whole orchestra towards a single point. Violinists turn their pegs by the smallest amount; brass players adjust with their breath; somewhere beside me, a string player winces, retunes, and tries again. It is one of my favourite moments in any rehearsal because it feels so reassuringly precise, as if sixty people are quietly converging on the same perfect answer. But that is also the strange part. The more I thought about what it really means to be "in tune", the more that certainty began to dissolve. The mathematics of tuning reveals something unexpectedly beautiful: a perfectly tuned musical system is impossible. What the orchestra is doing, then, is not finding perfection, but balancing competing imperfections so skilfully that, for a moment, they sound like harmony.

2. Hearing in Ratios

To understand why tuning becomes a mathematical problem at all, we first have to ask what a note actually is. When I play that opening A, I am not producing something mysterious or abstract. I am setting air into vibration, 440 times every second. That number is the note's frequency, its hidden numerical identity. Now suppose another instrument plays the A above it. The sound is clearly higher, yet it does not feel unrelated. Somehow, the ear recognises it as the same note in a different register, as though the two pitches belong to the same family. Mathematics explains why: the higher note vibrates at exactly twice the frequency of the lower one, giving the ratio 2:1 [1]. We hear that relationship as an octave. This is the first quiet miracle of music. What feels natural to the ear turns out to rest on an exquisitely simple piece of arithmetic. Long before harmony becomes emotional or dramatic, it begins as proportion.

3. Why Simple Fractions Sound Beautiful

The octave, though, is only the easiest example. Keep moving through music and the same numerical elegance appears again. A perfect fifth comes from the ratio 3:2, for every two vibrations of the lower note, the higher makes three [1]. It has a kind of openness to it, a steady confidence that makes it feel almost architectural, as though the two notes are holding each other upright. A major third, at 5:4, feels different: warmer, brighter, more settled, as though the sound has softened at the edges [1]. None of this is accidental. Intervals built from small whole-number ratios allow vibrations to align in remarkably orderly ways, which is why they so often sound consonant rather than tense. The more I looked at these fractions, the more seductive the idea became: if musical beauty seems to favour such clean mathematics, why not build an entire tuning system out of them?

4. The Temptation of Perfection

Once those simple ratios start to show up, the next thought seems almost unavoidable: why not trust them completely? The octave, the fifth, and the major third all sound beautiful because of how cleanly they relate to each other. So, the most beautiful tuning system would have to be one that is made up of all of them. It is a very tempting idea. You could picture a musical world where every interval is perfectly pure, every chord settles without any trouble, and harmony unfolds with the same certainty as maths. For a brief time, it seems clear that perfection is not only possible, but also clear. But this is where the dream gets dangerous. Numbers that sound beautiful in isolation do not always coexist peacefully inside a larger system. The ear may fall in love with purity, but the mathematics, stubbornly, asks whether purity can actually survive contact with the rest of music.

5. The Crack in the System

Mathematics, inconveniently, refuses to be charmed by beauty. Take the perfect fifth, that glorious 3:2 ratio, and imagine building an entire musical world out of it. Start on any note. Go up a fifth. Then another. Keep going until you have climbed twelve perfect fifths, and you might expect to arrive neatly back where you began, just seven octaves higher. It feels like the sort of symmetry music ought to reward. After all, twelve fifths and seven octaves seem to describe the same journey in different languages, but try the arithmetic and the spell breaks.

Twelve perfect fifths give:

$$\left(\frac{3}{2}\right)^{12} \approx 129.746$$

while seven octaves give:

$$2^{7} = 128$$

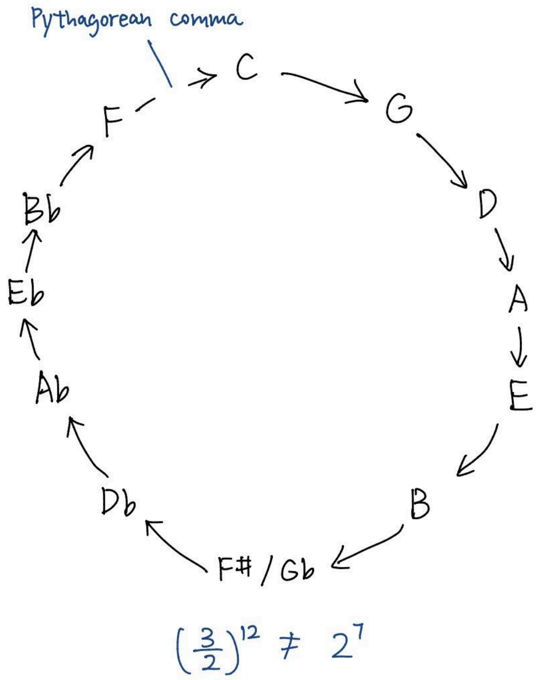

Figure 1. A sketch of the circle of fifths. Repeated pure fifths almost return to the starting note, but leave a small gap: the Pythagorean comma.

Those numbers are not equal. They are close, irritatingly close, but not close enough to save the dream. That tiny mismatch is called the **Pythagorean comma** [1]. The name sounds harmless, almost grammatical, yet it exposes something profound: the circle of fifths is not really a circle at all. It is a circle with a seam in it, a beautiful system that almost closes and then quietly fails.

6. The Problem with Pure Harmony

A mismatch of less than two hertz sounds trivial, almost petty, until you remember that music has to live, not just calculate. That tiny gap cannot simply vanish; it has to fall somewhere. If a keyboard is tuned to preserve beautifully pure intervals in one key, then another key must absorb the error, and when it does, the damage becomes audible. Chords that should sound calm and settled begin to shimmer in the wrong way, producing an uneasy beating, as though the notes are arguing under their breath. In extreme cases, musicians gave these ruined intervals a wonderfully dramatic name: **wolf intervals** [1]. The phrase is theatrical, but the problem is real. A tuning system built from perfect ratios can sound heavenly for a while, yet the moment the music wanders too far, the harmony starts to warp. This is the real consequence of the Pythagorean comma. It forces an uncomfortable choice: preserve perfect purity in a few places, or accept a little impurity everywhere so that music can move freely through every key.

7. Twelve Equal Steps

So musicians did what humans so often do when perfection refuses to cooperate: they compromised, but with extraordinary elegance. Instead of forcing some intervals to remain pure and letting one disastrous crack split the system open, they spread the impurity across the whole octave. Suppose each semitone is obtained by multiplying the frequency by the same constant, r. After twelve such steps, we must arrive exactly at the octave, which doubles the frequency, so:

$$r^{12} = 2$$

That means:

$$r = 2^{\frac{1}{12}} \approx 1.05946$$ [2][3]

This is the heart of **equal temperament**. Every note is reached by the same multiplier, not by a collection of perfect ratios, but by twelve equal slices of imperfection. It is a beautifully audacious idea. Rather than asking one interval to absorb the whole Pythagorean comma, we shave off a microscopic amount from every step, so no single key is ruined. The result is not purity, but balance. No fifth is perfectly 3:2, no third is perfectly 5:4, and yet every key becomes usable. The circle closes at last, not because the mathematics has been made flawless, but because the flaw has been shared fairly.

8. A Logarithmic Ear

There is a quieter elegance hiding beneath this compromise. In everyday life, we measure distance by addition: ten more metres means we simply add ten. But pitch does not work that way. To the ear, the journey from 220 hertz to 440 hertz feels like the same musical leap as the journey from 440 to 880, even though the second increase is far larger in absolute terms. What matters is not the difference, but the ratio: both notes have doubled, so both are octaves. Musical space, then, is not additive but multiplicative [4]. That is why equal temperament works by multiplying each note by the same factor, $2^{\frac{1}{12}}$, instead of adding a fixed number of hertz each time. In the language of mathematics, this is the quiet logic of logarithms. Equal temperament sounds natural not because it imitates perfection, but because it matches the way our ears already organise sound.

9. Why Music Must Compromise

And this compromise can be measured. In a perfectly pure system, a fifth should have the ratio:

$$\frac{3}{2} = 1.5$$

But in equal temperament, a fifth spans seven semitones, so its ratio is:

$$(2^{\frac{1}{12}})^7 = 2^{\frac{7}{12}} \approx 1.4983$$ [2][3]

That is astonishingly close to 1.5, yet not quite the same. The major third shows the sacrifice even more clearly. In pure tuning, it is:

$$\frac{5}{4} = 1.25$$

whereas in equal temperament it becomes:

$$(2^{\frac{1}{12}})^4 = 2^{\frac{1}{3}} \approx 1.2599$$ [2][3]

Again, close, but no longer pure. These differences are tiny enough that most of the time we accept them, and large enough that they matter. Equal temperament does not preserve the old perfect intervals; it replaces them with remarkably good imitations, carefully chosen so that no key is privileged and no modulation becomes impossible. That, to me, is the real beauty of the system. It admits defeat, but refuses disaster. Music keeps its freedom by giving up a little of its purity, and what we hear as harmony is, in part, the sound of that bargain holding.

10. How Far Off Is "Almost"?

If all of this still sounds like a tiny technicality, the compromise can be measured even more precisely. Musicians often compare intervals using cents, a logarithmic unit that divides one octave into 1200 equal parts. In this system, a pure fifth $\frac{3}{2}$ differ from an equal-tempered fifth $2^{\frac{7}{12}}$ by only about 1.96 cents [4], which is why the interval still sounds impressively stable. The major third is a more dramatic sacrifice: the pure ratio $\frac{5}{4}$ and the equal-tempered version $2^{\frac{1}{3}}$ differs by about 13.69 cents [4]. Suddenly, the compromise stops being vague. Some intervals are only gently nudged; others are more noticeably reshaped. Equal temperament does not flatten music into sameness. It distributes error unevenly, but strategically, preserving enough of the old purity that our ears still recognise beauty, while giving harmony the freedom to travel anywhere.

Interval	Pure tuning	Equal temperament	Difference
Perfect fifth	$\frac{3}{2} = 1.5000$	$2^{\frac{7}{12}} \approx 1.4983$	about 1.96 cents narrower
Major third	$\frac{5}{4} = 1.2500$	$2^{\frac{1}{3}} \approx 1.2599$	about 13.69 cents wider

11. The Beauty of Almost

When I hear that opening oboe A now, I hear it very differently from when I first joined the orchestra. Back then, it felt like the start of everyone trying to find one perfect note. Now it feels more like the start of a conversation. I play, the strings lean towards the sound, the brass adjust, and for a few seconds the whole orchestra is listening as carefully as it can, not to mathematics, but to one another. I think that is why this moment has stayed with me for so long. The numbers may refuse to line up perfectly, but the music begins anyway. No one in that hall is waiting for flawlessness. We are all making tiny corrections, giving things up, meeting each other halfway, and trying to create something beautiful that none of us could make alone. To me, that is what makes an orchestra tuning so moving. It is not the sound of perfection being achieved. It is the sound of people choosing, together, to make peace with imperfection and play on.

References

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