# Can you make 1-100 with just 2014?

**Inspired by Dr. Zye**

**Challenge: Can I make every number 1 to 100 with just the digits of my birth year, 2, 0, 1 and 4?**

***Well, that would be WAY too easy if I let myself have infinite successors [S(n)] or predecessors [P(n)]. So I'm setting a few boundaries for myself***

Rule #1: 1 successor/predecessor per expression.
Rule #2: No extra numbers (5, $\pi$, etc.). No concatenation.
Rule #3: Every digit must be used once.

***So with those boundaries set, I guess it's time to begin.***

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## Chapter 1: Just Zeroes, Ones and Addition

In the beginning, it's really easy. The set {2, 0, 1, 4} has a bunch of tiny naturals. 1 would be easy to make, we have a 1 already. But Rule #3 says we need to use every digit in the set. The 0 we have is really helpful in this case. We can multiply every number we don't need by 0, and the value doesn't change. So $1+0(2+4)=1$. And for 2, we could do the same thing. $2+0(1+4)=2$. And for 3, that's just 2+1. So let's do the same thing. $2+1+0(4)=3$. We can do this for 4, 5, and so on.

$4 = 4+0(1+2)$
$5 = 4+1+0(2)$
$6 = 4+2+0(1)$
$7 = 2+0+1+4$

Now, we need to get creative. We do not have any other digits to add, so we need to use 0 more efficiently. It's possible to turn 0 into a 1. $0!=1$. With this new discovery, we can do a lot more. $8 = 2+0!+1+4$. For 9, we could use a successor, but 9 is also $3^2$. Plus, we don't want to burn our successor. So, we can do $9 = (2+1)^2+0(4)$… but in that equation, we use two twos, one in the base and one in the exponent. Well, we didn't use our 4, so we can take the square root of it, that's two! So, we can rewrite our equation as $(\sqrt{4}+1)^2+0$. Then turn the 0 into a 1 via a factorial, $10 = (\sqrt{4}+1)^2+0!$, and that's how we get ten! But, we used all of our digits, and I don't want to use the successor just yet. So, we could make a more efficient 10. 10 is double 5, so we can do $2(4+1)$. Just add the 0!, and $2(4+1)+0!=11$. 12 is double 6, and 6 is just $4+1+0!$. So, $2(4+1+0!)=12$. For 13, we can use our successor around the entire expression.

S(2(4+1+0!))=13. 14 is double 7, so if we just shift the successor to the multiplicand, 2(S(4+1+0!))=14. 15 is triple 5, so we can make the 5 by adding 4 and 1, and create the 3 by adding 2 and 0 factorial. So, (2+0!)(4+1)=15. 16 is 4 squared, $4^2+0(1)=16$. Add the 1, $4^2+1+0=17$. Add 0!, $4^2+1+0!=18$. Successor, $S(4^2+1+0!)=19$. Okay, to survive the 20s, we need a new launchpad. We need to get into the 20s with just one digit. How is that possible? Well, we've been using the factorial only on 0, but we can use it on other digits. 4!=24. We're in the 20s! We need to subtract 4, and an extra 4 can be made by adding 2, 1, and 0 factorial. So, 4!-2-1-0!=20.

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## Chapter 2: Two Tens And a Little Bit More

We can take the factorial away from 0 so it doesn't subtract 1. 4!-2-1-0=21. Multiply the 0 by 1 to make that all 0, and 4!-2-0(1)=22. Now only subtract the 1, and multiply the 0 by 2 to get this expression, 4!-1-0(2)=23. We've already made 24, so skip it. For 25, we can turn the subtraction into addition, so 4!+1+0(2)=25. Add 2 for 26. 4!+2+0(1)=26. 27, another mellifluous number, because we can make an

equation in the natural order. $2+0+1+4!=27$. The beauty of math. Add 0 factorial, and $2+0!+1+4!=28$. Use the successor around the expression to make it 29. And finally, 30. 30 is just quintuple 6, or triple 10. So we can do (4+1)(2… uhm, we can't make 6 out of 2 and 0. Maybe we need to do triple 10 to get to 30. (2+1)(4… same problem. Wait, we're just adding 1 to the multiplier… we can do that with a successor! We can make the 3 by succeeding 2, $S(2)$, and now we need to make 10 out of 4, 1, and 0. Oh! That's perfect! Merge the 1 and ze-
*Rule #2: No extra numbers (5, $\pi$, etc.). No concatenation.*
Oh, I forgot I said "no concatenation" Now I regret that. But, it's not actually that big of a deal. We can bring in another function… the double factorial ($n!!$). The double factorial is like a factorial, but you skip every other number. If n is even, only multiply by even numbers less than n. If it's odd, only multiply by odd numbers less than n. Basically, $n!! = n\times(n-2)\times(n-4)\ldots$ until n reaches 2 if it's even or 1 if it's odd. And the double factorial of 4 is 4(2), so 8. 8+1 is 9… 9+0! is 10!
$S(2)(4!!+1+0!)=30$. But 31 is a really stubborn prime.

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## Chapter 3: Midway Crisis

So, we already used our successor. Well, we could try to make a more efficient 30 using less digits. 30=24+6 and we've discovered that 4!=24, and 6 is 3!. That 3 can be made by adding 2 and 1, so 30 is 4!+(2+1)! and we have a 0 left, and we've proven that 0 can turn into a 1. 4!+(2+1)!+0!=31. Three factorials in one equation. At least that's unlimited! We use a successor for 32. 33 is 32+1, but we used all our digits and our factorial. So we need to use more advanced operations. $2^5=32$, so we can make that 5 out of 4+1, and use the 0 to make a 1! $2^{4+1}+0!=33$. A successor makes 34. 35 is 36-1, and 36 is $6^2=36$ and we can make that while saving the 0 by using a successor. $S(4+1)^2=36$. Add 0 to keep 36, and add or subtract 0! to get 35 or 37. For 38, I... I can't seem to find an equation that uses the operations we've been using that equates to 38 while following my staying within our 3 boundaries. But who said we had to stick with these functions? Any function is valid! So I am going to draft in the Gamma Function.

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## Chapter 4: Drifting Far Away from Comfort

From this point on, there will be a ridiculous amount of advanced math symbols. The gamma function, defined as $\Gamma(x)=(x-1)!$, is really helpful to propel us through these next expressions. We need a new shortcut into the forties. 40 is $^{120}/_{3}$. We can make 3 with 2 and a successor. 120 is... $\Gamma(\Gamma(4))$. So $(\Gamma(\Gamma(4))/S(2))+0(1)=40$. -1, $(\Gamma(\Gamma(4))/S(2))-1-0=39$. -0!, 38. +1, 41. +0!, 42. For 43, we could use the 48 anchor. S(0!)(4!)=48. Now we need to make 5 out of 2 and 1... but how? Well, we need to draft in Uncle T! Wait, sorry, the Triangular Function ($T$). Triangular n ($T_n$) is 1+2+3...+n. It is the sum of n and every natural number below n. So $T_2+2=5$. But we only have 1 two and we need to make 2 out of 1 and 0 successors. How? Well, we must draft in Elegant Lady P! Sorry, *ahem* the Power Set ($\mathscr{P}$). In set theory, the Power Set is the set that encases all subsets including the empty set. If a set has 1 element, the power set has 2. Wait... I just accidentally made 2 out of 1! $|\mathscr{P}\{1\}|=2$. So $S(0!)(4!)+(T_2+|\mathscr{P}\{1\}|)=43$.

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## Chapter 5: Halfway There

But then, there's 44. There's no easy way to make it. We can bring the Triangular Function back and use it on 6. That would be 21, and 24+21=45. So 45 is constructible because 24 is just 4! and the 6 to make 21 is (2+1)!. Subtract 0! so 45 turns into 44, and $4!+T_{(2+1)!}-0!=44$. Then add 0! to get 46. Use the successor, 47. 48 is double 24, and 24 is 4!, so 2(4!)+0(1)=48. Add the 1, 49. Add 0!, 50. Successor, 51.

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## Chapter 6: No Giving Up

We must make a more efficient 50. We can't make 52 without it. 50 is just double 25, so 2(4!+1)=50. Now, we've saved a 0. Add the successor of 0!, and that's 2. As for 53, that's when it gets problematic. But, it's not actually a problem. We can get up to 55 with just one digit. $T_{T_4}$ is 55, with a nested triangular number. $T_4$ is 4+3+2+1, 10, and $T_{T_4}$ is 10+9+8+7+6+5+4+3+2+1. That's 55 down. But for 53 and 54, I don't think I have to explain how to make 2 and 1 with the numbers 0, 1, and 2, that is if the reader engaged in any time of schooling. Anyway, 53, 54, 56 and 57 are all constructible. And a successor makes 58. For 59, we need 60. 60 is half 120, and 120 is 5!. So (S(4)!/2)+0(1) is 60. Predecessor, 59. Add the 1 to make 61, add 0! for 62.

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## Chapter 7: Where It Starts To Get Bad

So now we need a NEW anchor. 60 with literally just 1 digit. It's possible... we need to bring in a new function. The tiny little whistl- uh, sorry. Sum-Of-Divisors function (σ). Sigma x is the sum of all integers divisible by x. And σ(24)=60. With this, we can make 63 by doing σ(4!)+(2+1+0). Add 0!, 64. Successor, 65. Now we make 6 with 2, 1, and 0. We did that with our elegant la- sorry, Power Set and Triangular Function. So, σ(4!)+($T_2$+|$\mathscr{P}${1}|)=66. Successor, 67. Now we need 8 with 2, 1, and 0. Well, |$\mathscr{P}${3}| is 8, so we can do |$\mathscr{P}${2+1}| to make 68 with the 4 used for the 60 and the 0... nothing. Add 0!, 69. Successor, 70.

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## Chapter 8: Prime Time

A wasteland with a premium name. Primes are ruthless in these challenges. We need a new anchor. (2+1)(4!)+0=72. Predecessor, 71. Successor, 73. Add 0!, 74. New anchor needed! (2+0!)(4!+1)=75. Successor, 76. We now need a NEW anchor. $T_{12}$ is 78, we can make that 12 with 4!/2. So the predecessor of $T_{4!/2}$ is 77. Successor, 79. Plus 1, 80. Successor, 81. Now, we need to make 4 without 4 or 2. $|\mathscr{P}\{2\}|=4$, so use 1 and 0! for 82. Successor, 83. Now, we need 84. That's quadruple 21. 21 is $T_6$, so make that 6 with (2+1)!, multiply by 4, waste the 0, and for 85, add 0!, and for 86, use the successor. We now need a new anchor. 91 is $T_{13}$, that 13 is (4!/2)+1. We need 87, 91-4. 4 with 0 is $|\mathscr{P}\{S(0!)\}|$, so 87 is constructible.

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## Chapter 9: Lightning Round

$T_{(4!/2)+1}$-S(0!)=89. Predecessor, 88. Bring that expression back, $T_{(4!/2)+1}$-S(0!), take the S away, $T_{(4!/2)+1}$-0!=90. 91 is already built, add 0!, 92. Successor, 93. 2 out of 0? Easy! $T_{(4!/2)+1}$+P(|$\mathscr{P}${S(0!)}|)=94. Take that less fancy P away,$T_{(4!/2)+1}$+|$\mathscr{P}${S(0!)}|=94. (|$\mathscr{P}${2)}|)(4!)+0(1)=96. Predecessor, 95. Successor, 97. Add the 1, 98! Add 0!, 99. 100 is $10^2$. 10 is $T_4$. So, $T_4{}^2$+0(1) is 100. We can even overshoot and make 101 and 102, but that's not the challenge.

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## Conclusion

It is indeed possible to make every number from 1 to 100 with just the set {2, 0, 1, 4}. It's difficult, but it's possible. Even overshooting too! You just gotta love math.