Mathematical Melodies

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When one thinks of the applications of maths, it is safe to say that the things that pop into mind would be data, finance, engineering, computer science, physical sciences, and economics. Today we think in terms of STEM people <u>or</u> creative people. There's a division, and this division is holding us back from reaching our full potential. Mathematics and science are seen as factual, serious subjects, while performing arts and creatives are seen as emotional, stereotyped as undisciplined, with the artists considered 'unstable' and 'poor'.

My goal for this essay is to attempt to shift that perspective, to show that the arts and the quantitative fields are not just of the same importance, but also walk hand in hand, where one cannot exist without the other -- the same way that fire cannot be lit without oxygen.

For example, origami provides a mathematical analogy for material folding, and origami techniques were used to create the "Miura-ori" pattern, which, in 1996, made the folding and deployment of solar panel arrays on the Japanese Space Flyer Unit possible.

Harry Kroto, a Nobel-prize-winning chemist used Buckminster Fuller's geodesic domes --a form of artistic architecture-- to determine the spherical structure of C-60 molecules, known as Buckminsterfullerene.

Microbiologists now use petri dishes as canvases, "painting" with colourful bacterial strains to understand microbial interaction and colony behaviour.

With my final example, let me also introduce my topic for this essay, by telling you that Pythagoras used string experiments to connect music to number theory in 500 BC, and that I will be using mathematics to introduce you to Indian music.

There are two forms of Indian music; Hindustani, from the north, and Carnatic, from the south. Carnatic classical music is more composition-based, and has stayed quite true to its original form, while Hindustani classical music is more improvisational in nature, and has been strongly influenced by Persian culture and evolved over time.

My main area of focus will be Hindustani music, where I'll talk about not only the simple, easy connections that can be made, such as the percussion section and the counting of the beats, known as *matras*, but also the involvement of sets and subsets, factorials for the formation and calculations in *taanas*, which are fast-paced, improvised note passages sung using the specific notes of a *raag,* which is what we call the melodic frameworks of Indian music.

Now, to state the obvious, a beat pattern in music is very heavily mathematical. Rhythm is the subdivision of a beat. Here come into play ratios and proportions, since the rhythm and beat show the relationship between a part and a whole, and are also divided into different sections.

Both the *Natyashastra* and the *Sangita-ratnaakara,* two texts on Indian performing arts, discussed problems in number theory still pertaining to modern mathematics, and also had applications to music. They considered some of these problems in rhythm, for example, understanding the number of ways you can break a 17-beat cycle into partitions of length 1,2,4, and 6. The answer, also shown in these texts, is 6236.

The rhythm in Indian music, also known as the *taal*, originated from the human heartbeat, setting it at around 72 beats per minute. The *taal* is cyclical in nature and in all the different types of *taal*, after the last beat comes the very first beat, and the cycle continues just like the pattern of day and night continues. The first *matra* in a taal is called the *sam,* and the way that Indian musicians think about *sam*, is that it is how one cycle flows into the next.

The distribution and measure of time also come from the concept of rhythm. For example, the intervals of minutes are equidistant, so the intervals of rhythm measurements are also equidistant.

Now, I would like to stress the importance of this rhythm, the *taal,* in Hindustani music. Pingle (1898) defined taal as: "Taal (time), which is an important factor in every system of music, regulates the relative durations of musical sounds, and as such seems to have considerably engrossed the Indian mind."
As a student of music, I can confidently say that the *taal* is just like time, keeping its pace while you have to work around it. The beat doesn't stop for you the same way that the minutes pass, and all you can do is catch up. I remember a competition where I had played the violin along with the *tabla,* which is one of more popular percussion instruments in Indian music, and in order to showcase my understanding of the beat, I had to play my piece while the tabla played different variations of the *taal*, in order to, in a way, hide the *sam,* which makes it harder to tell where the cycle is ending and from where I had to pick up the next line of my *raag.* To put it into perspective, imagine living on a desert island and having to follow an hourly routine without having access to any clocks. Yes, the night and day do give you some semblance of how much time is passing, but it sure makes it a thousand times harder!

The different kinds of *taanas* are just rhythms with a different number of beats. Some examples, along with the way they would be written are:

- Teentaal, the most common rhythmic cycle. This is a 16-beat cycle with the beats being divided into groups of 4. These divisions are known as *vibhaags.*

dha 1	**dhin** 2	**dhin** 3	**dha** 4
dha 5	**dhin** 6	**dhin** 7	**dha** 8
dha 9	**tin** 10	**tin** 11	**ta** 12
ta 13	**dhin** 14	**dhin** 15	**dha** 16

- Keharawa, an 8-beat pattern. It has eight *matras* in two equal *vibhaags.*

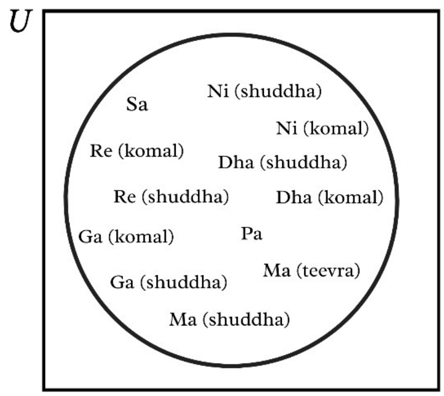
dha	**ge**	**na**	**tin**	|	**na**	**ke**	**dhin**	**na**
1	2	3	4	|	5	6	7	8

- Jhaptaal, a 10-beat taal with a 2-3-2-3 division pattern.

dhin	**na**	|	**dhin**	**dhin**	**na**
1	2	|	3	4	5
tin	**na**	|	**dhin**	**dhin**	**na**
6	7	|	8	9	10

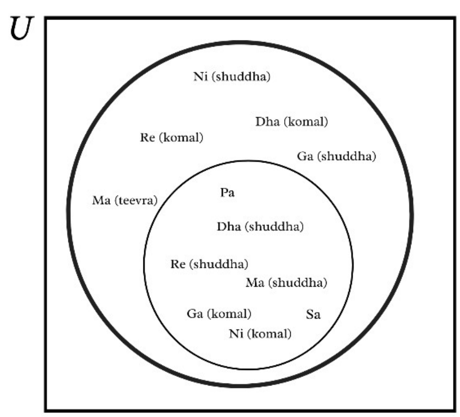
There are 12 notes, known are *swars*, in Indian classical music. *Swaras* are formed by combining the fundamental frequencies and their overtones.
In Western music, composition is often deemed sovereign, but in Indian music, for playing a *raag* all you need to know are the *swars* that fall under that *raag* category. Think of it this way:

A universal set is defined as a set containing all elements or objects under consideration for a specific, context-dependent problem. Our universal set, U, contains the 12 swars, which are-
Shadaj (Sa), Komal Rishabh (Re), Shuddha Rishabh (Re), Komal Gandharva (Ga), Shuddha Gandharva (Ga), Shuddha Madhyam (Ma), Teevra Madhyam (Ma), Pancham (Pa), Komal Dhavait (Dha), Shuddha Dhavait (Dha), Komal Nishad (Ni), Shuddha Nishad (Ni).



For our subset, which is our *raag,* I will take the example of my personal favourite *raag,* Raag Bhimpalasi. Bhimpalasi is an Audav-Sampurna, meaning it has 5 notes in ascent, 7 in descent. Yes, raags are also classified by the number of notes that you play during ascent and descent. Musical mathematics, am I right?

Putting that aside for now, let me show you the subset we would use for Bhimpalasi:



The Bhimpalasi notes for ascent, i.e. aaroh, are Sa, Komal Ga, Shuddha Ma, Pa, Komal Ni.
The notes for descent, i.e. avroh, are Sa, Komal Ni, Shuddha Dha, Pa, Shuddha Ma, Komal Ga, Shuddha Re.

Raags also have *vadi* and *samvadi* swars, where vadi is the most dominant, frequently used *swar*, and samvadi is the second most prominent *swar.* For Bhimpalasi, Shuddha Madhyam is the *vadi swar*, and Shadaj is the *samvadi swar.*

Bhimpalasi's *thaat* is Kafi. This classification method is for organising raags based on their parent scale. It is a theoretical framework from which raags are made. A thaat also only consists of an *aaroh*. The different thaats help musicians identify the structural framework and swaras within a raag, helping with recognition. To further put that into perspective, there is a universal set, *U*, containing all the 12 notes. There is a subset containing all the notes in a thaat, *T*. The subset for *T* is *R*, the raag derived from that *thaat*. This can also be represented mathematically as:

$U \supseteq T$, and $T \supseteq R$

Matang Muni in Brehdeshi (around10th century CE) explains the formation of 5040 Tanas from the seven notes of the scale. The seven notes mentioned would be the five *Shuddha* notes, along with *Shadaj* and *Pancham.* 5040 is a very large number, is it not?

To better explain this, let us take each note as a letter, and talk about the different numbers of combinations that can be made as we take more letters, i.e., notes.

Number of Notes	Notes	No. of Combinations	
2	a, b	2	2!
3	a,b,c	6	3!
4	a,b,c,d	24	4!
5	a,b,c,d,e	120	5!
6	a,b,c,d,e,f	720	6!
7	a,b,c,d,e,f,g	5040	7!

e.g. For a, b, c, and number of combinations being 6, the combinations would be abc, acb, bac, bca, cab,cba

This is a demonstration of the calculation of a number of different combinations which can be formed with any natural number, in our case the different notes. This means that the total number of combinations that are possible with n number of different notes is equal to n multiplied by the number of combinations formed by (n-1) number of notes, which is equal to n multiplied by (n-1) multiplied by no. of combinations with (n-2) number of notes… and so on. If we extend this equation further and further, we get n x (n-1) x (n-2) x (n3)...... x 1 which is mathematically the same as n factorial.

Another application of maths in music, is of microtones and ratios, where the 22 *shruti* (microtones) in Carnatic music are derived from frequency ratios, for example, 3:2 and 4:3, focusing on intonation.

I think it is extremely interesting that <u>all</u> fields of music, including the Western melodic patterns, the Hindu raga, the Japanese pentatonic scale, etc. are bound to mathematically derived codes.

This connection of mathematics and music is also extremely helpful to the young students learning both subjects. I was introduced to Hindustani music quite early on, first with singing classes at the age of five, and then, violin classes from the age of around eight. I don't know if it is just my love of the subject that has caused me to see mathematical patterns so frequently in music, or if it really is that obvious, but the fact is still true, that music and maths are intertwined and are built off of each other.

Another fun fact for you: The music curriculum for Indian music is based on the original historical classical music. Indian music and mathematics have both originated from the *Vedas*, which are the earliest body of Indian scripture, and lay down the basis of Hinduism. The interesting part is that the *Vedas* show that there exists a close relationship between Indian music and mathematics. *Raags* have roots in the Vedic period, in which everything was heavily based on a scientific foundation. Vedic literature analysed *swaras* using complex mathematical combinations, essentially creating different combinations of notes to form different ragas.

Shankar Mahadevan is a famous Indian singer, music director and composer. He also holds an engineering degree in Computer Science from Mumbai University, India.
"I never really connected the dots between music and maths," Shankar Mahadevan said, "But maybe learning to play the harmonium and veena before I was five, helped develop my mathematical skills without my realizing it.'

"Composing music is like problem solving-you have to put text and rhythm together," a faculty member at Peabody Prep said. "Musical rhythms are mathematical equations, like 4/4 or 2/4 time. You use addition and subtraction skills: How many more beats do you need? How many more ways can we divide it? Music also helps children develop language. A musical phrase is like a verbal sentence. It has a beginning, a middle, and an end; and the inflection changes in different parts of the sentence.

To end this essay, all I would like to say is that mathematics is the basic framework for so many things, and that you would not have music in all its glory without maths, and you would also not have a lot of the breakthroughs in maths if music had never existed. It is foolish to believe that these two are separate when they are just two shades on the same spectrum, even if different colours, two books by the same author, even if different genres, and are soldered by the same gold, even if one is an elegant piece of jewellery and one is a mighty sword.

References & Citations

www.nsf.gov

https://www.ghimindia.com/

https://digitabla.com/reference/compositional-forms/theka/

https://madrasmusings.com/vol-25-no-19/musing-on-music-and-maths/

https://ww.shankarmahadevanacademy.com/

https://artiumacademy.com/

Guha, Smita (2022) "Mathematics in Indian Music: Examining Children's Learning Process," Journal of Global Awareness: Vol. 3: No. 2, Article 4.

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