Brute Strength Approach:
Study hours and the Premature Graying of Hair

(What's the mathematical relationship?)

1 Introduction

Whilst I was participating at a STEM event set at school, I noticed the sheer amount of students roughly my age with an inordinate amount of gray hair pouring out of their heads. I was curious after the event and did some simple research online and discovered that higher stress levels could lead to more gray hair.

While it is scientifically proven that higher cortisol levels could lead to more gray hair (Bryant, 2020), research papers have never attempted to show a clear mathematical relationship between study hours and premature graying of hair.

With all that said, allow me to humour you through attempting to plot out the relationship of study hours and amount of gray hair on a simple and clear cartesian plane.

2 Assumptions

Figure. 1: Adrian

(Ramos, 2026)

I. Meet Adrian, he studies A-levels and lives in Hong Kong; he's your typical STEM student. He enjoys the subjects he studies but suffers from academic stress. We will take Adrian's tolerance to studying as a default for the clear graph. It must be understood that more study hours doesn't directly translate to higher stress levels as there are still individuals that don't experience as much academic stress as others

II. Adrian gets a good night's sleep every night – to reduce cortisol levels and keep the relationship between study hours and stress levels as 'ideal' as possible

III. Adrian lives in a happy family and has lots of friends at school – following up with the trends of assumption 2, reducing external stress so the essay can completely be focussed on academic stress

IV. The subjects/topics that Adrian are studying are really difficult for his academic ability but all the questions roughly remain at the same level of difficulty, he's studying for his final exams – explains the stress part from assumption 1, questions of high difficulty lead to the brain triggering the "fight-or-flight" response which leads to increased stress ("Understanding the Stress Response in Exams - What Happens in Your Brain?", 2023)

V. Adrian's study schedule is horrible – he doesn't take any breaks to let his brain relax at any point

So in essence, making academic activities the sole contributor to Adrian's rise in stress levels.

3 The Maths

I've always liked to see things as rates of change, it allows for a far simpler and easier-to-visualise-with environment to 'get things going' . An example would be sketching out the displacement, velocity and acceleration graphs when just given one of the graphs, differentiate or integrate it and voilà you get the graph you want. To apply the rate of change ideas into this essay, I will find the rate of change of stress levels wrt study hours ($\frac{dL}{dt}$), and the rate of change between concentration of gray hair wrt stress levels ($\frac{dg}{dL}$) as I am attempting to plot out the relationship between study hours and gray hair with study hours being the x axis and the concentration of gray hair being the y axis.

The two derivatives ($\frac{dL}{dt}$, $\frac{dg}{dL}$) would allow me to use related rates to find $\frac{dg}{dt}$ which can then be integrated in terms of dh, ultimately leads to the pretty cartesian plane illustration.

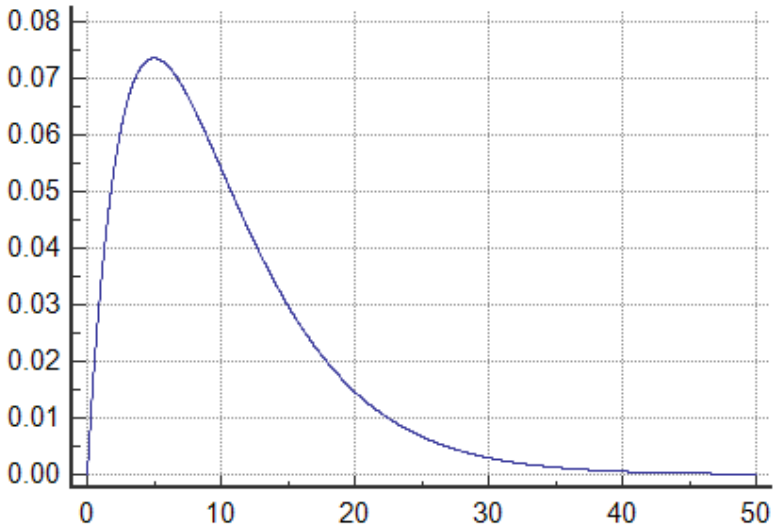
To start off, I must point out a clear mathematical relationship between study hours and stress levels for Adrian. While there is also no clear mathematical formula for showing the relationship of these two variables, research papers have gone to a much further extent to attempt to find a mathematical relationship of stress vs hours of study than gray hair vs hours of study.

From the paper "Relation between stress, time management, and academic achievement in preclinical medical education: A systematic review and meta-analysis", the relationship between stress and study hours has positive correlation, with more studying leading to more stress and with distress increased

roughly linearly with study duration, and even 1–2 hours of intense studying was associated with higher distress compared with no studying (Khajeali et al., 2021). It is quite reckless, however, to assume that the mathematical relationship between duration of studying and stress for Adrian to be linear, since the repetition of studying difficult topics over and over again for sustained periods of time which could lead to brain (or cognitive) fatigue, also known as one of the main sources of the induction of stress.

The way stress levels can be measured is through cortisol levels (replacing L for stress levels from now on), when a person gets stressed, cortisol is released into the bloodstream. The graph for cortisol levels will be a gamma distribution curve (probability density function, PDF), which looks like this:

Figure. 2: Gamma Distribution Curve



(Schoonjans, 2025)

The reason that there is a local maximum is because cortisol levels will not just keep on increasing and increasing but rather peak at some point, when too much mental load is stacked overtime from studying, it could lead to Adrian having a burnout which decreases cortisol levels. A burnout however leads to extreme mental, physical, and emotional exhaustion (Mental Health UK, 2024) which could affect the efficiency of study but that's not really what i'm looking for is it?

I will just set the maximum time for Adrian's duration of study as 16 because he still gets his 8 hours of good sleep.

The x axis and the y axis of the gamma distribution curve are time (hours) and cortisol level respectively.

Since this IS a maths essay afterall, I will talk more about why the gamma distribution curve works and can be implemented to get fitting results for the cortisol level vs time graph. Gamma distribution curves are used in cortisol research and models often treat cortisol secretion as a stochastic process (Brown et al., 2001) which in other words is the mathematical collection of random variables, usually indexed by time.

It works because as mentioned earlier, cortisol is not released as a smooth, steady stream; it is released in bursts by the HPA axis, circadian rhythm and feedback control. The gamma-shaped kernel is a good way to model the rise and decay shape of each burst. Based on the study schedule of Adrian, he most likely has around 4 cortisol spikes per day.

Where the equation can be shown by

($t \in [0, 24]$, for time)

$$L(t) = L_0 + \sum_{i=1}^{4} A_i g(t - \tau_i)$$

With:

L_0 being the baseline cortisol

τ_i are the four spike times

A_i being the spike heights

$$g(u) = u^{k-1} e^{-u/\theta}, \; u \geq 0$$
$$g(u) = 0, \qquad u < 0$$

With:

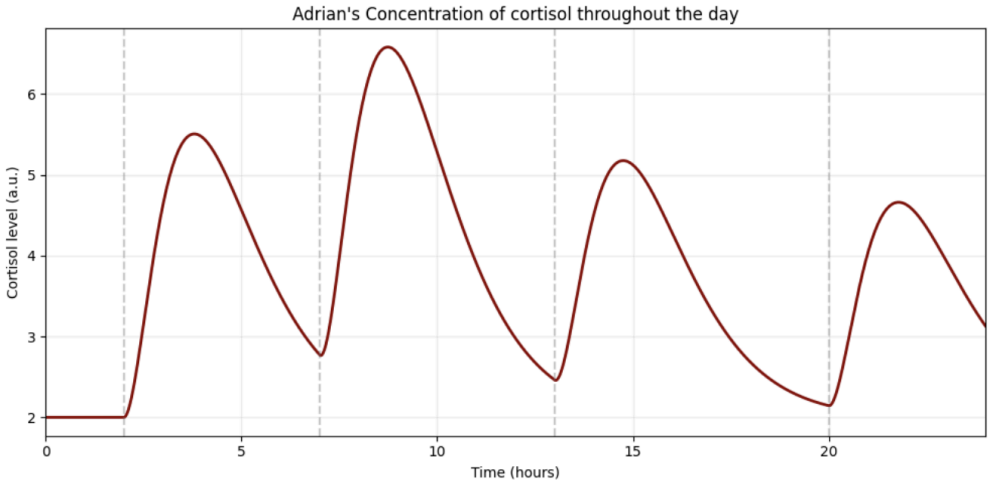
$g(u)$ being the gamma shaped pulse

$u = t - \tau_i$, being the time since the onset of the i-th spike, measured in hours

k being shape parameter (often called the shape or order of the gamma distribution)

θ being the scale parameter controlling how fast the spike decays after the rise

Figure. 3: Adrian's Concentration of cortisol throughout the day



The values won't be discussed because they are hypothetical. Figure.3 is but a rough model.

(I am just attempting to make it similar to an actual cortisol vs time graph)

Now to find the derivative:

$$\frac{dL}{dt}(t) = \sum_{i=1}^{4} A_i g'(t - \tau_i)$$

When $u = t - \tau_i \geq 0$:

$$g'(u) = \frac{d}{du}(u^{k-1} e^{-u/\theta})$$

$$g'(u) = u^{k-2} e^{-u/\theta}(k - 1 - \frac{u}{\theta})$$

Therefore:

$$\frac{dL}{dt}(t) = \sum_{i=1}^{4} A_i (t - \tau_i)^{k-2} e^{-(t-\tau_i)/\theta}(k - 1 - \frac{t-\tau_i}{\theta}),\ t \geq \tau_i$$

Now to find $\frac{dg}{dL}$ (derivative of conc. of gray wrt cortisol levels). The concentration of gray hair is directly proportional to the damage to melanocyte stem cells driven by sympathetic-system activity ("Solving a Biological Puzzle: How Stress Causes Gray Hair", 2020). Assuming that cortisol levels is directly proportional to the damage to melanocyte stem cells, I can find $\frac{dL}{dt}$ (size of melanocyte stem-cell pool in hair follicles, smaller = worse = more gray hair) and $\frac{dg}{dt}$ and use related rates to find $\frac{dg}{dL}$ (jeez im starting to get tired of derivatives).

$$\frac{dL}{dt} = -\ A(t)\ -\ \beta stress(t)S(t)$$

With

$A(t)$ being the intrinsic aging term, it should theoretically be 0 since Adrian isn't above 20 years old yet but Adrian's workload makes it a non-zero value

$\beta stress(t)$ being stress-driven stem-cell loss rate

$S(t)$ being the size of melanocyte stem-cell pool in hair follicles at time t

$$\frac{dg}{dt}\ (t)\ =\ \alpha B(t)e^{(-\lambda C(t))}$$

With

α being sensitivity of graying to stress

B being stress-induced burst-like damage term (also could be modelled as gamma)

$exp(-\ \gamma C(t))$ being the diminishing returns ~ once many hairs are already gray, extra stress adds less visible gray

$$\frac{dg}{dL} = \frac{dt}{dL} * \frac{dg}{dt}$$

$$\frac{dg}{dL} = \frac{\alpha B(t)e^{(-\lambda C(t))}}{-A(t)-\beta stress(t)S(t)}$$

Finally,

$$\frac{dg}{dt} = \frac{dg}{dL} * \frac{dL}{dt}$$

$$\frac{dg}{dt} = \frac{\alpha B(t)e^{(-\lambda C(t))}}{-A(t)-\beta stress(t)S(t)} * \sum_{i=1}^{4} A_i(t-\tau_i)^{k-2} e^{-(t-\tau_i)/\theta}(k-1-\frac{t-\tau_i}{\theta})$$

$$\frac{dg}{dt} = \sum_{i=1}^{4} A_i(t-\tau_i)^{k-2} \frac{\alpha B(t)e^{((-\lambda C(t))-[(t-\tau_i)/\theta])}(k-1-\frac{t-\tau_i}{\theta})}{-A(t)-\beta stress(t)S(t)}$$

*Note: (SUPER ROUGHLY PUT TOGETHER)

Now to integrate it…

$$\int dg = \int (\sum_{i=1}^{4} A_i(t-\tau_i)^{k-2} \frac{\alpha B(t)e^{((-\lambda C(t))-[(t-\tau_i)/\theta])}(k-1-\frac{t-\tau_i}{\theta})}{-A(t)-\beta stress(t)S(t)})dt$$

However, can be represented more clearly by going back to S as technically it's the main cause for gray hair anyways:

$$g(t) = \int_{t_0}^{t} \sum_{i=1}^{4} [\frac{A_i g_i'M(s)}{D(s)}]ds$$

Where (this is the last time, i swear!)

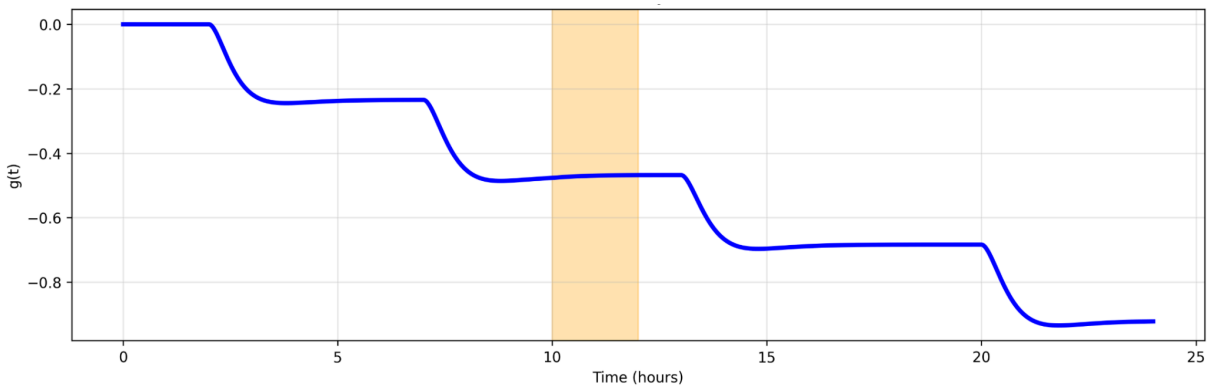
$$g_i'(u) = u^{k-2} e^{-u/\theta}(k - 1 - \frac{u}{\theta})$$

$M(s) = B(s)e^{(-\lambda C(s))}$ is the gray hair modulation

$D(s) =- A(t) - \beta stress(t)S(t)$

$$g(t) = \sum_{i=1}^{4} \int_{t_0}^{t} [A_i g_i'(s - \tau_i) * \frac{M(s)}{D(s)}]ds$$

Figure. 4: g(t) vs t



4 Conclusion and Evaluation

As seen on the graph, the longer Adrian studies the more, the more his melanocyte stem cells size decreases which means that he grows more gray hair. As seen on the graph, the relationship between stem cell size and time is NOT directly proportional under a constant increase in brain load (studies). Instead, it acts just like the cortisol graph, with stem cells decreasing in bursts.

A potential takeaway from this final result is that breaks should be taken every time $g(t)$ starts to slope so that cortisol can decrease which allows for stem cells to lose as little size as possible, resulting in less gray hair. If your study/work schedule is similar to Adrian's then it's time to change it

5 Bibliography

Bryant, Erin. “How Stress Causes Gray Hair.” National Institutes of Health (NIH), 3 Feb. 2020, www.nih.gov/news-events/nih-research-matters/how-stress-causes-gray-hair.

Ramos, Giuseppe. “A Black Silhouette of a Man Standing Icon Isolated.” Vecteezy, 2026, https://www.vecteezy.com/png/55364763-a-black-silhouette-of-a-man-standing-icon-isolated. Accessed 6 Apr. 2026.

“Understanding the Stress Response in Exams - What Happens in Your Brain?” OxfordAQA International Qualifications, 3 Oct. 2023, www.oxfordaqa.com/news/understanding-the-stress-response-in-exams-what-happens-in-your-brain/.

Rosenberg, Ayelet M, et al. “Quantitative Mapping of Human Hair Greying and Reversal in Relation to Life Stress.” ELife, vol. 10, 22 June 2021, https://doi.org/10.7554/elife.67437.

Khajeali, Nasrin, et al. “Relation between Stress, Time Management, and Academic Achievement in Preclinical Medical Education: A Systematic Review and Meta-Analysis.” Journal of Education and Health Promotion, vol. 10, no. 1, 28 Jan. 2021, p. 32, pmc.ncbi.nlm.nih.gov/articles/PMC7933620/, https://doi.org/10.4103/jehp.jehp_600_20.

Schoonjans, Frank. “Gamma Distribution Functions PDFGamma, CDFGamma and RndGamma with Graphs and Online Calculator. | MedCalc.” MedCalc, MedCalc Software, 24 Aug. 2025, www.medcalc.org/en/manual/gamma-distribution-functions.php. Accessed 11 Apr. 2026.

Mental Health UK. “Burnout.” Mental Health UK, Mental Health UK, 2024, mentalhealth-uk.org/burnout/.

Brown, Emery N, et al. “A Stochastic Differential Equation Model of

Diurnal Cortisol Patterns." AJP Endocrinology and Metabolism, vol. 280, no. 3, 1 Mar. 2001, pp. E450–E461, https://doi.org/10.1152/ajpendo.2001.280.3.e450.

"Solving a Biological Puzzle: How Stress Causes Gray Hair." Harvard.edu, 2020, www.hsci.harvard.edu/news/solving-biological-puzzle-how-stress-causes-gray-hair.