Why are hexagons the jack of many trades of geometry?

Tom Rocks Maths Essay Competition

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Overview

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Look closely at the world around you and you might start noticing a pattern. Bees build honeycombs made of hexagons. Snowflakes grow with sixfold symmetry. Many 3D printers fill objects with hexagonal patterns. Even aircraft panels often contain structures that look suspiciously like honeycomb.

At first, this might seem like a coincidence. But when the same six-sided shape keeps appearing in both nature and engineering, it starts to look less like a coincidence and more like a pattern worth investigating. In fact, you could say that hexagons keep popping up from every angle.

So why does this shape appear so often?

The answer turns out to be surprisingly mathematical. Hexagons appear whenever systems are trying to solve an optimisation problem: how to use the least material while enclosing the most space and distributing forces efficiently.

By the end of this essay, the hexagon will look less like an ordinary polygon and more like a shape that is, quite literally, well-rounded for the job.)

1 The geometry of a hexagon

A regular hexagon has six equal sides and six equal interior angles. Each angle is 120°. This can be calculated using the formula for the interior angle of a regular polygon: $((n-2)180)/2$ For a hexagon n=6: $((6-2)180)/2 = 120$ This number is more important than might first appear. If three hexagons meet at a single corner, their angles add up to: 360

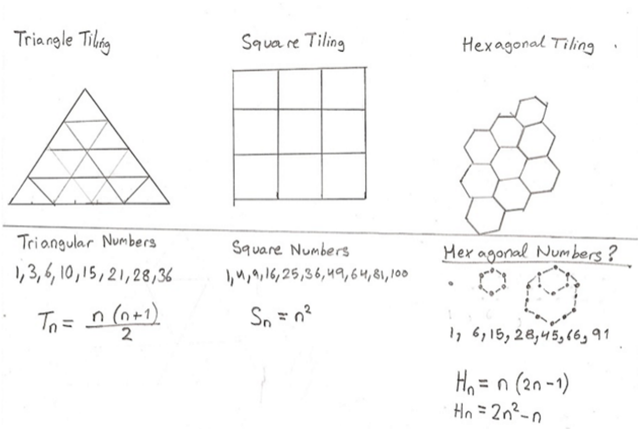

Figure 1: Result of tiling triangles, squares and hexagons

This means hexagons can fit together perfectly without leaving gaps. This leads to an interesting mathematical question: which regular polygons can tile a flat surface? Surprisingly, there are only three: equilateral triangles, squares , regular hexagons Other shapes don't work. Regular pentagons, for example, have angles of 108°, which refuse to fit together neatly to make 360°. In other words, pentagons simply don't make the right angles for the job.

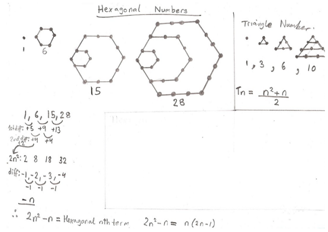

Figure 2: Hexagonal Numbers

Hexagonal numbers are closely related to triangular numbers. In fact: "Hexagonal number"=2×("Triangular number")-n Even more surprisingly: Every hexagonal number is also a triangular number. For example: 6 is triangular 15 is triangular 28 is triangular So hexagons are hidden triangles. This is unexpected. A six-sided shape secretly shares a pattern with a three-sided one.

2 Efficiency

Dividing a surface into many equal regions happens in several real-world situations: bees storing honey, engineers designing lightweight materials ,3D printers filling the inside of objects The goal is usually the same, which is to maximise the space inside each region while minimising the material used to create the wall / divider. This is now a problem involving area and perimeter. We want the largest possible area enclosed with the smallest possible boundary. We want to maximise: Area / Perimeter There is a geometric result related to this: for a perimeter, the shape that encloses the largest area is a circle. Unfortunately, circles come with a small problem. They refuse to tile the plane neatly. If you try to cover a surface with circles, gaps always appear between them. Even circles can't get their act together when it comes to tiling. So while circles are the most efficient shape individually, they cannot be used for repeating patterns.

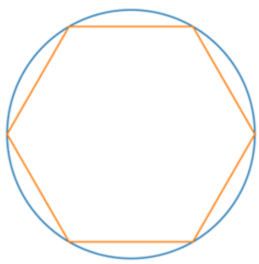

Figure 3: Hexagon and Circle Similarity

Circle = best possible (but unusable for tiling)

Hexagon = best practical alternative This creates a mathematical compromise. We need a shape that: tiles perfectly , encloses as much area as possible , uses minimal perimeter Out of the shapes that tile perfectly (leaving no gaps), hexagons turn out to be the best possible shape

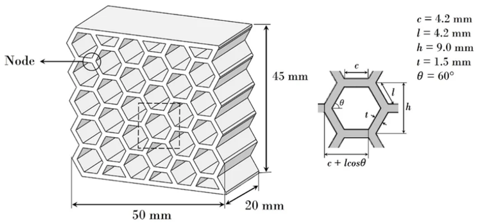

Figure 4: Honeycomb Structure

In 1999, mathematician Thomas C. Hales proved the Honeycomb Theorem, which shows that a hexagonal grid divides a surface into equal areas using the least total perimeter. So, bees have been solving a complex geometry problem for millions of years. You could say their solution is the bee-st possible one.

3 Structural Integrity

Hexagons are not only efficient at dividing space but also excellent at distributing forces.
Forces travel through materials along different paths. When a load is applied to a hexagonal grid, the forces spread across multiple directions.
At each corner of a hexagon, three edges meet at angles of 120°. This symmetry allows forces to distribute evenly across the structure.
Squares contain long straight lines that bend more easily. Triangles are extremely strong but require more material.
Hexagonal patterns provide a compromise between strength and efficiency. They distribute stress effectively while keeping material usage relatively low.
You might say hexagons help engineers, keep everything in balance, from every angle.

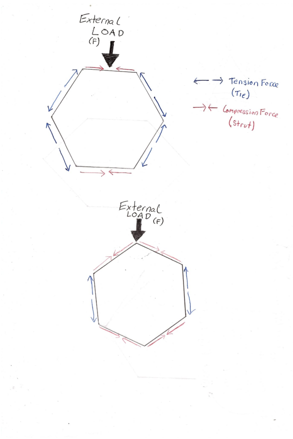

Figure 5: How a force is distributed on a hexagon

This sketch shows how a hexagon responds to an external load applied from above. The force (from the external load) is transferred through the edges of the shape, creating a mix of compression and tension forces. The red arrows indicate compression (struts) and blue arrows indicate tension (ties). This shows that the force does not remain at the point of application but is redistributed around the hexagon. This spreading of forces across multiple edges helps reduce the load on any single edge, demonstrating how hexagons can efficiently distribute stress, even though they still experience both tension and compression at different points.
Structural integrity can be quantified using stress: S =A/F

Where:

S = stress

F = applied force

A = area over which the force is distributed

For a single shape the force is transmitted through its edges.
Triangle:force is distributed across 3 edges
Hexagon:force is distributed across 6 edges
Assuming equal distribution:
Force per edge (triangle) =F/3

Force per edge (hexagon) =F/6
This shows: Each edge in a hexagon carries less force than in a triangle However, more edges = more points for deformation
So, triangles are the best shape for handling/ distributing a force, hence there is triangulation. However, hexagons are not too bad, they are still able to handle / distribute a force. Just slightly worse than triangles.

4 Structural Rigidity

A key concept in structures: Rigid structure condition: $2n-3$
Where n= number of joints.

Triangle $(n=3) : 2(3)-3=3$ (exactly rigid)
Hexagon $(n=6) : 2(6)-3=9$(not inherently rigid)
What does this mean?

Triangles are inherently rigid Hexagons require additional support or tiling to prevent deformation

Shape	Edges	Force per Edge	Rigidity
Triangle	3	$F/3$	Very High
Hexagon	6	$F/6$	Moderate
Honeycomb	Many	$F/(6N)$	High(Collective)

Table 1: Comparison of structural integrity

5 Practical Uses

In engineering honeycomb structures are super helpful in aerospace. They are strong and light like in the mirrors of the James Webb Space Telescope.

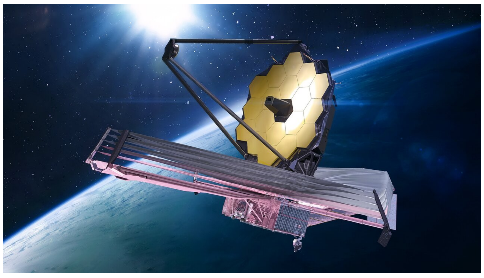

Figure 6: James Webb Space Telescope

People also use hexagons in furniture and design because they fit together well.
Squares and rectangles are more popular, for packaging.
They line up perfectly in rows without any empty space.
Hexagons are useful. Squares and rectangles are just easier to use.
Hexagons have their advantages though.

6 Practical Constraints vs Geometric Advantages

Hexagons have some properties when it comes to geometry. To be honest they can be a bit of a pain to make.
1) They often make a lot of waste when you cut them out
2) They need tools that are more complicated than what you need for squares.
Triangles are really good for building things because they are super strong and don't bend easily.
Rectangles are great for storing and moving things around.
Hexagons still have their uses like in honeycomb structures, where its really important to spread out forces evenly and be efficient.
Overall hexagons are pretty good, at doing a lot of things even if they aren't always the choice to work with.
They are still used in cases because of their benefits. Hexagons may not be the sharpest shape in one area, but they certainly cover all sides.

7 Conclusion

Hexagons can be considered as the "jack of all trades" as they consistently perform well on many properties. They tile without gaps, distribute forces through interlocking structures, and have a strong area-to-perimeter efficiency compared to other polygons. While they are not the best in any category such as triangles are stronger and circles are more efficient. But they achieve a good balance of each property.

They are used in both nature e.g Honeycomb and advanced engineering e.g the James Webb Space Telescope,. However, practical constraints like manufacturing difficulty and material waste limit their usage compared to simpler shapes like squares and triangles.

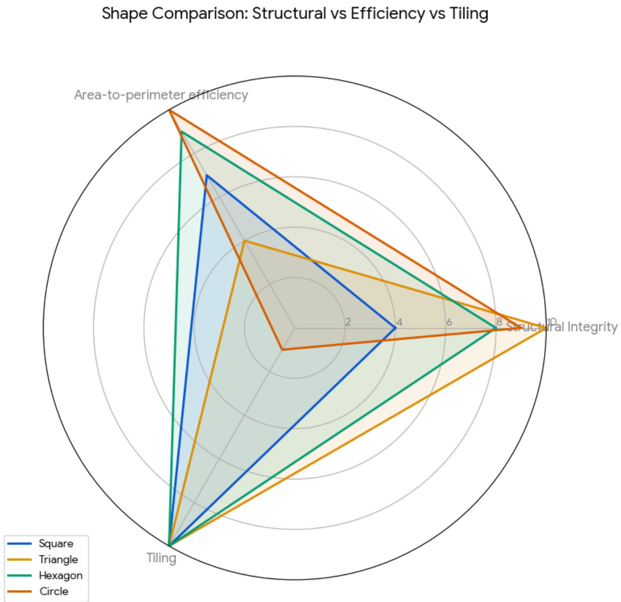

Figure 7: Radar Graph

Overall, hexagons stand out not because they are perfect in one area. But because they are effective across many. This balance of properties justifies them as a "jack of all trades" in geometry.
This demonstrates that in mathematics and real world applications, the most effective solutions are not always those that are the best at a single property, but it is rather those which achieve the best overall balance.