

Why is most of the global wealth owned by a small proportion of the population?

1 Introduction

Global wealth is significantly concentrated. According to the UBS Global Wealth Report, the richest 1% (those with more than \$1 million) own 47.5% of the world's wealth, which is approximately \$214 trillion. Conversely, adults with less than \$10,000 account for roughly 40% of the world's population. Furthermore, the world's twenty-six richest billionaires own a crazy \$2.872 trillion as of 2023. This is greater than the most nations' GDP, according to World Bank GDP data.

2 Pareto distribution

A pareto distribution is a statistical distribution that models phenomena where a small proportion of occurrences account for majority of the effect. This applies to global wealth because a large proportion of wealth belongs to a small proportion of the population and vice versa.

Pareto distribution is defined by two parameters:

- Scale parameter (x_m) - the minimum value of the pareto distribution. All values must be greater than or equal to x_m
- Shape parameter (α) – how unequal the distribution is and how “thick” the tail is.

Higher values of α (e.g. 3-4) represent a distribution with fewer large values (thinner tails as the tail decays faster). In the context of wealth, higher values of α demonstrates that extreme wealth is rarer and that there is a lower wealth inequality. On the other hand, lower values of α (e.g. 1-2) represent a heavy-tailed distribution where extreme wealth is more common and wealth inequality is higher.

$$P(X > x) = \left(\frac{x_m}{x}\right)^\alpha \text{ for } x \geq x_m$$

The above formula calculates the probability that a random variable X exceeds a given value x.

As x (wealth) increases, the probability of finding someone with that wealth decreases following a power law (will be explained later). The rate of decrease is controlled by α .

A key component of the pareto distribution is the 80/20 rule. The 80/20 rule in our context would be the top 20% holding 80% of the total. For this to be the case, α must equal 1.161. However, realistically, empirical estimates usually place α between 1.5 and 2.5 for the top tail. This means that extreme wealth is statistically common relative to what a normal distribution would predict, and that wealth is more concentrated (e.g. the top 10% holding 70-80% of wealth in many countries, such as Saudi Arabia).

Wealth follows a pareto distribution at the top because:

- Wealth often grows exponentially rather than from fixed amounts especially for the multimillionaires and billionaires e.g. returns from investments. This increases wealth inequality.
- Those with higher levels of wealth have access to better opportunities to further expand their wealth. This “rich get richer” idea mathematically generates a power law tail (again will be explained later)
- Wealth has no limit unlike biological traits such as height allowing extreme values of wealth to appear

A crucial point to clarify is that the entire wealth distribution doesn't follow the pareto distribution. The pareto distribution only applies to the upper tail. The middle class follows a log-normal distribution, where wealth is earned mainly from savings, labour income and home equity. The distribution comes about from multiplicative shocks (e.g. promotions, lengthy periods of saving and home price appreciation) combined with restrictions that prevent extreme wealth to occur. These restrictions exist because the system of labour income doesn't allow extreme wealth to be acquired.

In addition, inequality within the middle class differs from inequality at the top. A middle-class household at the 90th percentile of the middle class could have double the wealth of a household at the 50th percentile. However, in the pareto distribution, a billionaire could have thousands more wealth than someone whose wealth is equal to x_m .

Statistical modelling also shows a structural break at the top 10% of wealth. Below this point, wealth follows log-normal distribution and above this point, a power law emerges and wealth can grow exponentially.

3 Power Laws

A power law is a mathematical relationship where a change in one variable leads to a proportional change in another variable, raised to some exponent.

$$y = kx^{-\alpha}$$

Where:

- y is the number of observations above a minimum size
- x is the minimum size threshold
- k is a constant
- α is the exponent that determines how quickly the relationship decays

Applied to the wealth context, the number of people with wealth greater than a specified amount decreases as a power of that amount.

In a power law, the tail decays slowly (polynomially). However, in log-normal distributions (how middle-class wealth is modelled statistically), the tail decays rapidly (exponentially). This polynomial slow decay makes extreme wealth statistically common rather than impossible statistical anomalies. Power laws are characterised by

heavy tails so extreme values occur far more commonly than in normal distributions and outliers are fundamental to the distribution, not anomalies. A Pareto distribution is an example of a power law.

Another specific type of power law is Zipf's law where $\alpha = 1$. In a Zipf distribution, the second-largest value is half the size of the largest, the third largest is one-third, and so on. If wealth followed Zipf's law exactly, the richest person would have twice the wealth of the second richest, three times the third, etc. In reality, α is approximately between 1.5 and 2.5 for wealth. This is steeper than Zipf so the gap between the wealthiest and second wealthiest is larger than what Zipf's law would predict it to be.

Power laws emerge from:

- Multiplicative growth (Gibrat's Law) – growth occurs through multiplicative effects (e.g. investment returns) rather than addition (e.g. savings) so a power law tail is naturally developed. Over time, the inequality between those who get higher growth rates and those who get lower growth rates expands exponentially.
- Cumulative advantage – economic phenomenon where initial advantages (such as wealth, status, popularity and talent) accumulate over time, allowing people who possess early advantages to gain disproportionately more than those with little to no early advantages. Therefore, the probability of acquiring new wealth is proportional to initial levels of wealth. This is because wealthy individuals have access to: better investment opportunities (e.g. hedge funds and private equity), political influence and sophisticated tax planning and avoidance to name a few. This creates a vicious cycle that mathematically creates a power law.
- Wealth has no upper bound so power law tail can expand infinitely
- Behavioural factors also amplify power laws. Wealthy individual can take greater risks. This allows them to pursue high-return and high-risk investments (e.g. venture capital) that can increase wealth drastically. However, less wealthy individuals don't have a safety net and must prioritise financial stability. Furthermore, the wealthy also have access to experienced advisors who focus on returns and reduce taxes, further increasing wealth inequality.

However, some researchers argue that the wealth tail isn't a perfect power law tail but a power law with an exponential cutoff. These researchers would therefore argue that wealth can't expand infinitely but that the Pareto tail "bends" at extremely high wealth levels. This exponential cutoff can be modelled by $e^{-\lambda x}$. The exponential cutoff prevents infinite wealth development. On the other hand, there are debates as to whether the exponential cutoff is real.

A key disadvantage of the power law model is that wealth data is difficult to fully measure. This can be due to offshore wealth, unreported income and valuation challenges (e.g. private firms). Therefore estimates of α can be inaccurate which can lead to wrong evaluations of wealth inequality.

4 Conclusion

It can be argued that wealth inequality is mathematically inevitable due to the way that our financial system works. Billionaires aren't statistical outliers but expected products of the system. In order to decrease wealth inequality, we must adapt our financial system. Changes can include:

- Progressive taxes (taxing the wealthy at higher rates) as higher tax rates tend to increase α and therefore reduce concentration of wealth
- Estate taxes, reducing the initial advantages that wealthy people have and therefore reducing cumulative advantage. Inheritance is a massive multiplicative shock and allows the power law tail to persist across generations. Therefore, inheritance tax will have a considerable impact on reducing wealth inequality.
- Financial regulation – affecting access to high-return investments which reduces multiplicative shocks for wealthy individuals