Stabbing at π: Buffon's Needle and the Art of Accidental Math

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1 Introduction

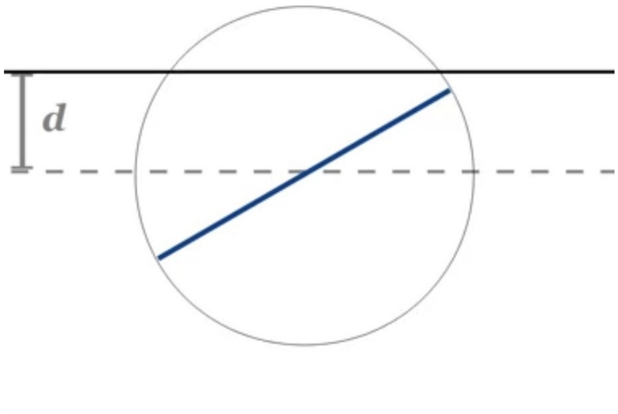
It is days before π day and you've been invited to the π day dinner party(can you guess what will be served?). Everyone coming has been assigned to come up with a formula for π or a problem that includes π. However, wanting to be creative and being a π enthusiast you decide to come up with a party trick. You heard a while ago of a trick involving ratios and π, perfect for the occasion! The trick in mind? Buffon's needle which, if you know anything about this you will know, the host will hate you for doing this due to the clean up. The basis of the problem is that if needles of a certain length are dropped onto a table with lines evenly spaced double the length of the needle apart, then the number of needles dropped divided by the number needles touching the line equals π as the number of needles dropped tends towards infinity.

2 The day before

But you can't just turn up and perform this trick. Any mathematician knows that first, you must convince yourself that this is going to work and how it will work. To do this we must formulate a proof and do a test run. For this test run and proof you take your infinitely long table and lay infinite thin strips of paper with the gap between each piece of paper being a length 2L (L being a scalar quantity). Then, you grab your bag of infinite, straight needles all of length L which are to be dropped onto the table. You do not drop them yet though, as first we must prove it would work. For if this didn't work and we dropped the needles it would be a costly mistake as it would be a lengthy process picking them all up; forever some may say. Now we have our basics so what next?

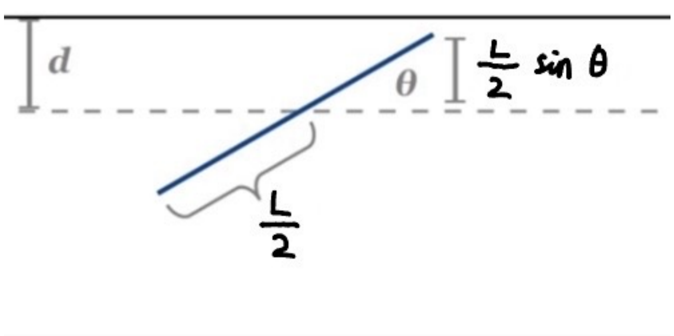
2.1 Landing of the needle

The needle could land anywhere on the table and can land in any orientation. The orientation of the needle is what creates the circle where π will appear. This circle is formed as the possible points each end could be on the table forms a circle. The image below shows an example circle of points this needle could have landed plus a distance from the center of the needle to the closest paper strip and a normal to the distance from the center of the needle to the thin strip of paper. The diagram below shows a circle of points that the ends of the needles could be located whilst the center remains fixed.



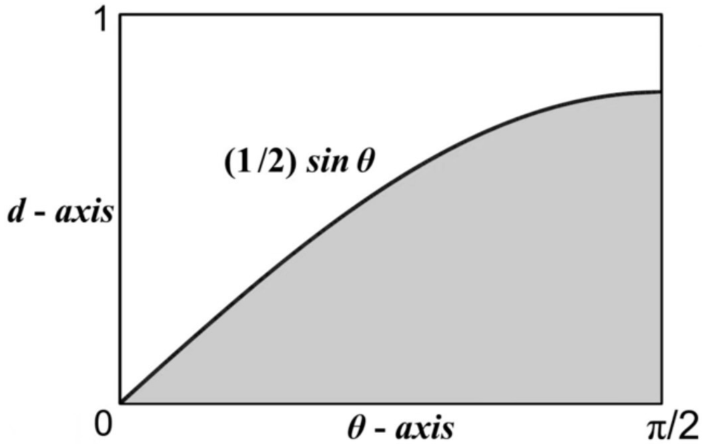
Now you ask the question, what is the probability of the needle intersecting one of these thin lines of paper? Following from our previous diagram we can say that the distance from the center of the needle to the closest line is defined as 0 $\leq$ d $\leq$ L. The needle also makes an acute angle θ with the normal, this is defined as $0 \leq \theta \leq \frac{\pi}{2}$.

Using trigonometry we can see the distance from the center of the needle to the paper is $\frac{L}{2} \sin \theta$. The diagram below shows this.



3 Proportions

After grabbing some food you return and now its less than 24 hours before the party. This means we must work fast if we want to be sure this will work by the time of the party. The needle can only intersect the line of paper if d $\leq \frac{L}{2}\sin\theta$. We can graph d against θ an we get this graph:



For the calculations we know the ratio of the needles length to the distance between the paper strips is 1:2. So we can assume L = 1 for our calculations giving the needle a length of one unit and the distance between paper strips two units. The distance from the center of the needle to the closest strip of paper is now $\frac{1}{2}\sin\theta$. Now, the probability of a needle crossing the paper is given by

$$Probability = \frac{Area\,under\,curve}{Area\,of\,rectangle}$$

$$Area\,under\,curve = \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin\theta d\theta$$

$$Area\,under\,curve = \frac{1}{2}[-\cos\theta]_0^{\frac{\pi}{2}}$$

$$Area\,under\,curve = \frac{1}{2}$$

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$$Area\,of\,rectangle = \frac{\pi}{2}$$

$$Probability = \frac{1/2}{\pi/2}$$

$$Probability = \frac{1}{\pi}$$

Now we can see that the total number of needles that intersect the paper over the total needles is 1 over π. You now feel confident that this will work. Now we have shown that for a bag of N needles the number of successes (x) you can do:

$$\frac{N}{x} = \pi \qquad , as\ N \to \infty$$

So now ready to rest for tomorrow you get into bed and try to sleep (hard to do the the excitement of π day being tomorrow) but then an idea comes to mind. A late night revolution of another proof you remember hearing of. So like any mathematician at 2am you get up to prove and test this new idea. You also remember you forgot to even see if the trick will work in reality so you open the bag of needles and let loose. Infinite needles coat the infinite table and you count up the needles and it has worked! The ratio is exactly π. But you are determined to ensure that this will work leaving nothing to chance. Time to perform a final proof.

4 Barbier's proof

Time is of the essence as the party is only half a day away so we must move fast. We keep our conditions the same but we assume the needle can intersect the lines of paper multiple times and we model this as a discrete random variable. This gives the expected number of intersections as:

$$E = (1)p_1 + (2)p_2 + (3)p_3 + ...$$

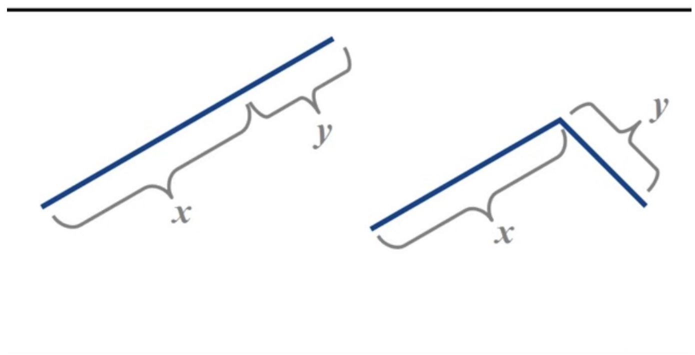
This is where p_k is the probability of the needle intersecting k times. For a needle of length 1 we can have at most 1 intersection so $p_2 = p_3 = ... = 0$. Therefore E(1) = p_1. We also define E(L) as the expected number of intersections for a needle of length L.

5 Out of needles

You then realize that you are out of needles as you used them all up in the previous test.So, you go to the infiney Store (open 24/7/∞) but they only have infinite bags of needles of random shapes and sizes. No straight needles of length one. You think for a while but come up with a solution; so you grab a few bags of needles of different shapes and sizes. Once home however, you find one straight needle left in the other bag you used earlier. You use this needle of length 1 for the start of your proof.

5.1 Splitting the needle

We can now divide the needle into 2 parts x and y. The total number of intersections of this needle can be given by $E(x + y) = E(x) + E(y)$ by the linearity of expectation. This holds for bent needles too, so you take a bent needle and label its two parts x and y (see figure below for comparison of needles).



Now for needles of length n we can calculate E(n) as E(n) = E(1+1+....+1) for n amount of ones. This gives;

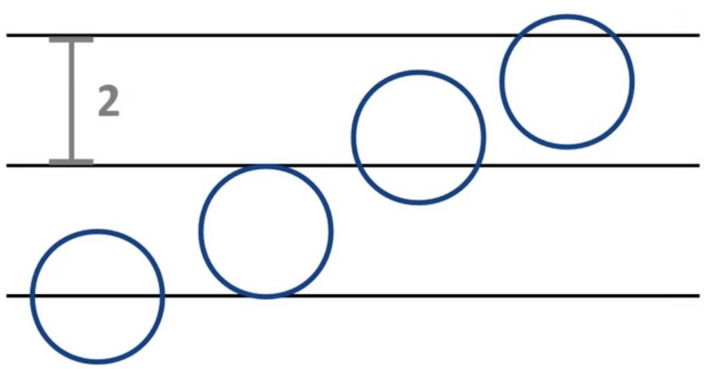
$$E(n) = nE(1)$$

we can generalize this further for needles in the shape of a polygon with perimeter x (where x is a rational number) giving;

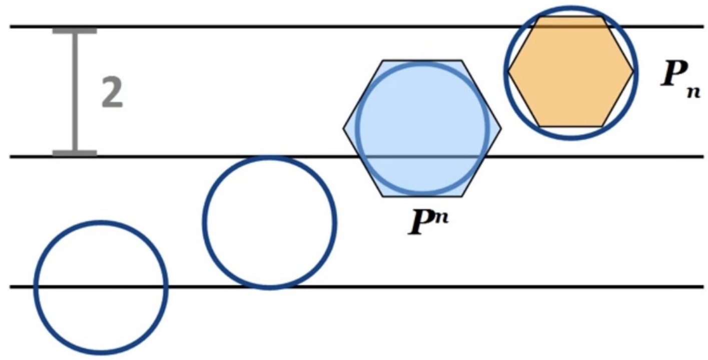
$$E(x) = xE(1)$$

6 Going round in circles

After hours of testing needles and only a few hours to go until the party, you feel hopeless and ready to just bake a pie instead until you see a strange needle catch your eye. A circular needle of radius 1. Perfect! This means the circle will have a diameter of 2 and, as seen in the figure below, will always intersect or touch the lines of paper twice.



Now, how can we relate this to needles of polygonal length? Well, we can have an inscribed polygon (P_n) and a circumscribed polygon (P^n).



The expected value of intersections of the circle can be bounded by these polygons such that;

$$E(P_n) \leq E(circle) \leq E(P^n)$$

But we know the expected number of intersections of a circle is always 2 so;

$$E(P_n) \leq 2 \leq E(P^n)$$

However, we can use our previous fact that for a polygon of perimeter x the expected number of intersections is just the perimeter times E(1). But as n tends to infinity, the perimeters of P_n and P^n tend to 2π so this gives;

$$2\pi E(1) \leq 2 \leq 2\pi E(1)$$

$$2\pi E(1) = 2$$

$$E(1) = \frac{1}{\pi}$$

So again we have proven that the probability that a needle of length 1 intersecting a line is $\frac{1}{\pi}$. This is the same conclusion as our first proof so it must be correct! Finally, we are done just in time with only two hours to spare.

7 Conclusion

However, you have spare bags of infinite needles of many different lengths and sizes so you take the needles of length one (infinitely many of them) and test one last time to be sure your correct. Once again the test works so you finally conclude that for N needles with x intersections, as N tends to infinity $\frac{N}{x}$ approaches π.

This shows how π is found in many unexpected places and shows the importance of π across mathematics and is one of, if the greatest discovery in mathematics and is applicable across the vast field. The question of Buffon's Needle was proposed merely as a simple question but it returned a beautiful answer linking probability, geometry and π for one of the first times. It is also considered the first Monte-Carlo style experiment, long before technology was a thing.

But now you realize you have no time to dwell as there are infinitely many needles to pick up and only one hour until the party. But that is a problem for next years competition.