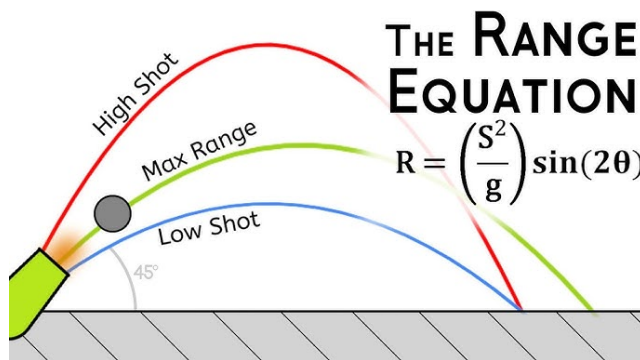


The Hidden Mathematics of the World Wars

The World Wars, a pivotal turning point in the history of warfare due to sheer scale of technological advancements and industrialisation. However, beneath the visible signs of warfare such as trenches, artillery and tanks lay a less visible but equally important force: mathematics. Even though not always acknowledged in combat, mathematics played an important role in most areas of the World Wars, from calculating where the next artillery shell would land to breaking the coded messages of one's enemy. This essay will consider the extent to which mathematics played a hidden yet crucial role in World War One.

Mathematics and artillery: the science of projectile motion.

The first great application of mathematics during the World Wars involved the determination of shell trajectories. Where artillery fire could be directed with some degree of accuracy toward its target, artillery shells required mathematical computation as their flight paths were highly irregular. This made gunnery an exercise in applied mechanics.



One way in which mathematics was applied to the World Wars was through equations like the range equation, which models the motion of projectiles and allows the distance a shell will travel to be predicted from its initial speed and launch angle. This is given by the equation

$$\text{Range} = (\text{initial velocity squared} \times \sin(2 \times \text{angle})) \div \text{gravitational acceleration}$$

From this, it can be observed that there are three main factors that determine how far the projectile will travel. First, the initial velocity is squared, which indicates that a small change in velocity produces large changes in the range of the projectile. Second, the angle at which the projectile was projected has an influence on its range, as indicated by the $\sin(2\theta)$ factor. The range would therefore be greatest when the angle is 45° , as the sine of 90° is one. From this, it is also clear that a range can be produced by two different angles due to the symmetry of the sine function. Third, the acceleration due to gravity appears in the denominator, which indicates that the greater the acceleration due to gravity, the lower the range of the projectile. In relation to the World Wars, this equation made artillery calculations more precise, making the act of firing almost mathematical.

Rates of change and uncertainty in warfare

During a battle like Verdun, there is an application of mathematical thought in the estimation of casualty rates, which is made implicitly by military strategists. There was no certainty about the

final result, but there were expectations based on rates of loss of life over a certain period of time. This is equivalent to rate modeling because there is an accumulative increase in casualties depending on the duration and frequency of battles, represented mathematically by $C'(t)$. The rate of change (dC over dt) is influenced by several factors including the number of artillery, strength of defense, and concentration of troops. This also demonstrates that there is mathematics even without calculus. On top of that, strategic plans could also involve expected value, where the outcomes involved probabilities and not certainties.

Based on what I have said above, it can also be said that casualty predictions in combat such as in the Battle of Verdun do not only depend on a particular moment but vary dynamically, where the casualty rate changes continuously based on the battle's situation. As such, when representing the casualties with the letter C , the relationship between C and t does not follow the linear function, making $C'(t)$ dependent on time. As such, if there is an increase in the use of artillery, the rate will increase, which corresponds to an increase in the magnitude of the derivative. Conversely, with better defensive systems, there would be a reduction in the casualty rate, thereby resulting in a decreased magnitude of the derivative. As such, the above example corresponds to a variable differential equation in a manner where the derivative varies due to different variables. This allows military commanders to adapt to the unpredictability of warfare by being able to estimate the casualty rate at different parts of the battle by taking into account different variables.

The reason why I brought up the Battle of Verdun as an example is because it clearly shows how miscalculating these rates of changes can lead to deadly consequences. To sum up the battle quickly, Verdun was a key defensive and iconic city located in France along the Western Front which the German Commanders knew the French would give up due to its cultural significance. Due to this the German commander Erich von Falkenhayn thought he would be able to draw out the majority of the French army and inflict a massive amount of casualties to them in the siege using tactics like bombardment and starvation draining France's resources and leading to Germany winning this war of attrition. However Erich von Falkenhayn deeply underestimated the rate of casualties that the Germans would sustain leading to the battle being won by the French with a huge amount of casualties on both sides. Showing how not understanding mathematical rate of change is detrimental in warfare