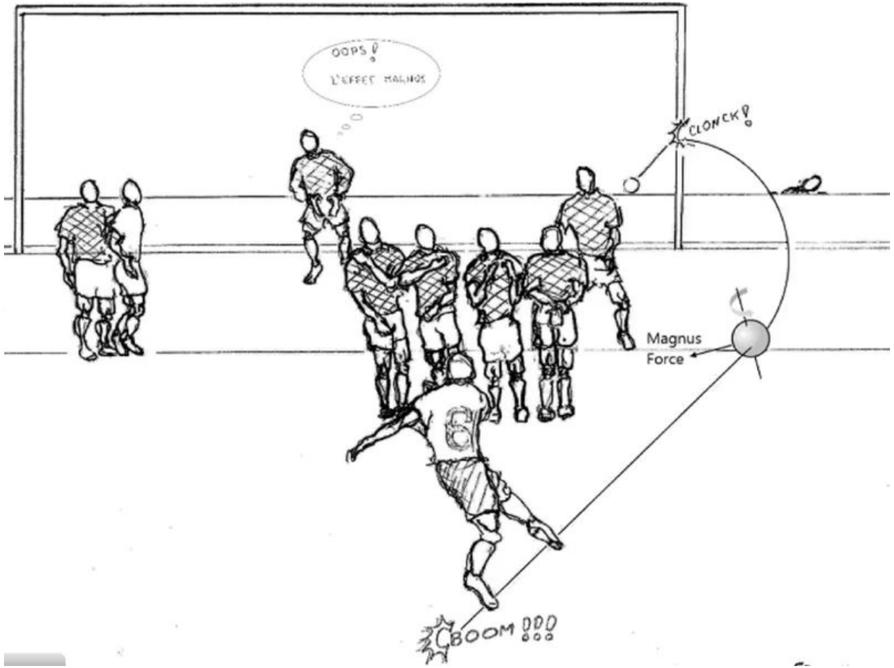

Can any mathematician
score a free kick?

What are the chances of scoring a free kick…?

Let's take the top player: Ronaldo

- He has taken over *700* direct free kick attempts (taken over 300 at Real Madrid only)
- Estimated to have scored *65* goals
- His success rate is to be around *9.3%* compared to the average *6.7%* an average footballer does
- Hitting the ball at top speeds of *100 km/h* (just over *62.1 mph*)
- Mastering the iconic ‘knuckleball’ technique
- Is it all just practice and skill or is there some maths and physics behind it?

What do you need for the perfect free kick?



1. **Angle and trajectory:**

   Understanding angles and curves helps with bending the ball around the wall

2. **Spin and physics:**

   The curved path of the ball is explained by the magnus effect

3. **Optimisation:**

   Choosing the best spot to aim can be thought of as a geometry problem

1) Angle and trajectory:

When a free kick is taken (ignoring spin and air resistance initially);

Ball kicked at a speed of v_0, *Horizontal angle θ*, *Position at (x,y)*, At a time of t, *g= 9.81ms^-2*

$$x(t) = v_0 \cos\theta\, t$$

$$y(t) = v_0 \sin\theta\, t - \frac{1}{2} g t^2$$

Bending around the wall :

The wall prevents the ball go in straight path towards the goal but rather curve around the call to get to the goal which introduces *lateral displacement z(t)*

$$\frac{d^2 z}{dt^2} = \frac{F_{\text{Magnus}}}{m}$$

For a non spinning kick to go over the wall the minimum angle could be estimated by:

$$y(t_{\text{wall}}) \geq h_{\text{wall}}$$

$$v_0 \sin\theta_{\min} \cdot t_{\text{wall}} - \frac{1}{2} g t_{\text{wall}}^2 \geq h_{\text{wall}}$$

The *average height being 2m* and $t_{\text{wall}} = \frac{d_{\text{wall}}}{v_0 \cos\theta_{\min}}$

2) Spin and Physics (The Magnus Effect):

The magnus effect is a ‘physical phenomenon where a spinning object moving through a fluid’ for this instance air;

$\vec{\omega}$ *= Angular velocity of the ball (rads^-1)*

$\vec{v}$ *= Velocity vector of the ball*

S = Proportionality constant which is dependant on air density and ball radius

$$\vec{F}_{\text{Magnus}} = S(\vec{\omega} \times \vec{v})$$

For a ball spinning about the vertical axis (sideways):

$$F_{\text{Magnus}} \approx \rho\, r^3\, \omega\, v$$

p = air density (≈1.2kgm^-30

r= ball radius (≈0.11m)

w = spin rate

v = ball speed

The lateral acceleration:

$$a_z = \frac{F_{\text{Magnus}}}{m}$$

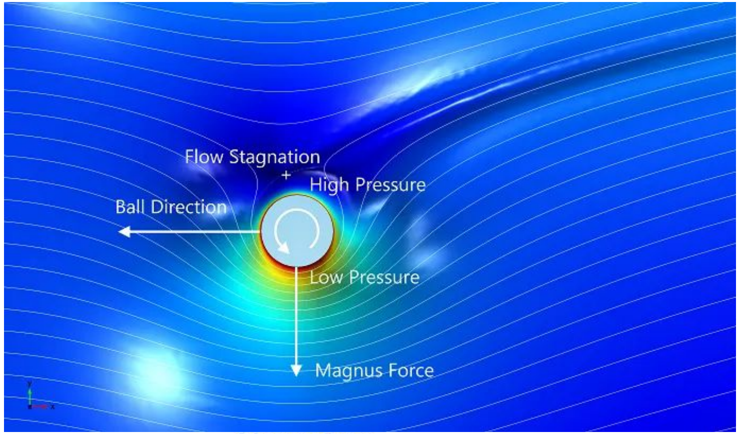
The lateral displacement z(t) is obtained by integrating the acceleration:

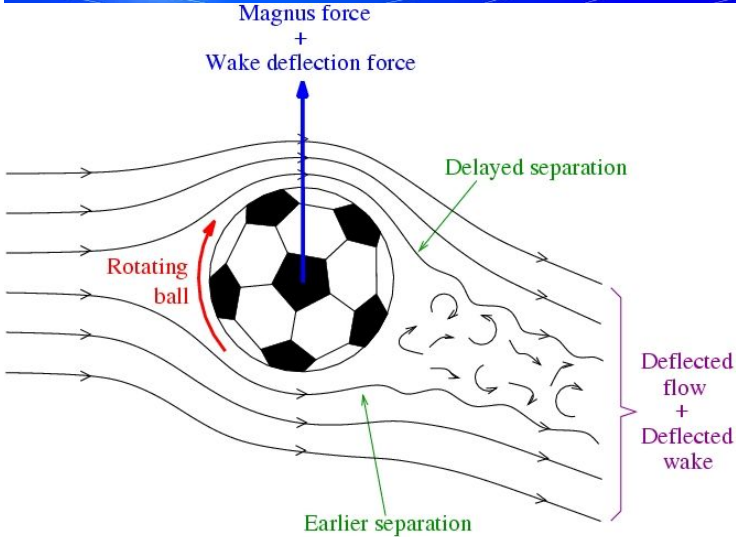
$$z(t) = \int_0^t \int_0^{t'} a_z \, dt' dt$$

Faster spin (w) means larger lateral acceleration and therefore sharper bend

High velocity (v) means stronger magnus effect but shorter flight time therefore smaller total deflection

- Ronaldo's knuckleball minimises *w* which reduce spin but created chaotic air flow which leads to an unpredictable lateral motion





3) Optimisation:

Using optimisation we can aim to maximise the chance of scoring by selection specific target points (x_{goal}, y_{goal})

To clear the wall:

$$y(d_{\mathrm{wall}}) \geq h_{\mathrm{wall}}$$

To say within the goal at a distance:

$$0 \leq y(d_{\mathrm{goal}}) \leq H$$

$$0 \leq z(d_{\mathrm{goal}}) \leq W$$

Minimise the goalkeepers reaction:

The goalie covers a certain height and lateral range so you would need to aim outside this region either one of these three ways

$$y(d_{\mathrm{goal}}) > h_{\mathrm{keeper}}$$

$$z(d_{\mathrm{goal}}) > w_{\mathrm{keeper}}$$

$$z(d_{\mathrm{goal}}) < 0$$

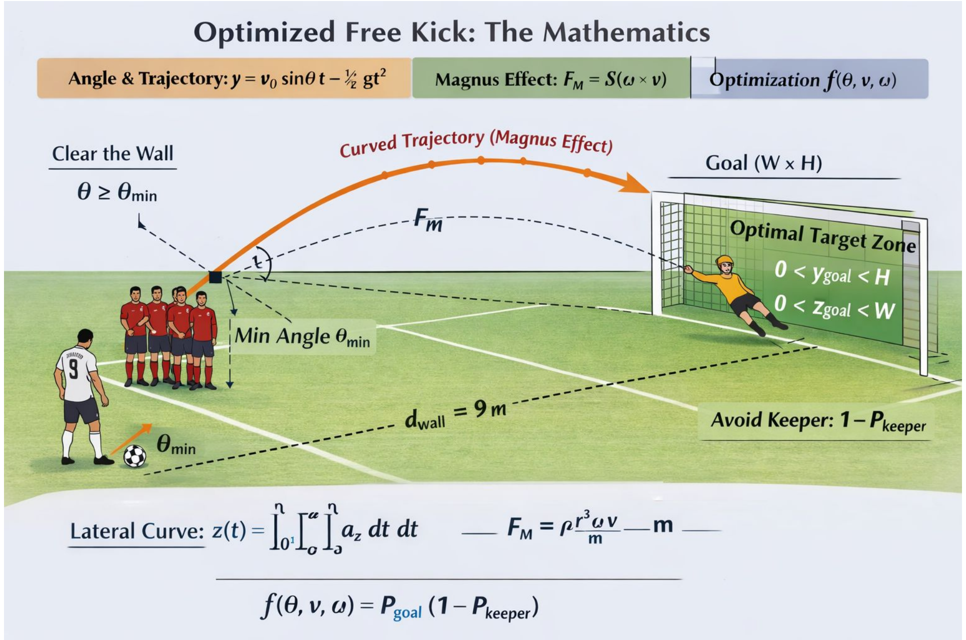
Mathematical formulation;

Let the total scoring "*score*" be a function *f(θ,v,ω):*

$$f(\theta, v, \omega) = P_{\text{ball in goal}}(\theta, v, \omega)$$
$$\cdot\,(1 - P_{\text{keeper save}}(\theta, v, \omega))$$

The optimal free kick is obtained by solving:

$$(\theta^*, v^*, \omega^*) = \arg\max_{\theta, v, \omega} f(\theta, v, \omega)$$



Example using Ronaldo’s realistic numbers

<u>Assumptions;</u>

Distance to the goal: d = 25m

Wall distance: 9m

Wall height: 2m

Ball speed: v0 = 28ms^-1

Launch angle:

Ball mass: m = 0.43kg

Ball radius: r = 0.11m

Air density: p = 1.2kgm^-3

Spin: w = 8 rads^-1

1. Can it clear the wall;

Horizontal motion:

$$x = v_0 \cos\theta\, t$$

Time to reach the wall:

$$t_{wall} = \frac{9}{28\cos 18^\circ}$$

$$\cos 18^\circ \approx 0.95$$

$$t_{wall} \approx \frac{9}{26.6}$$

$$t_{wall} \approx 0.34\, s$$

The height of the wall:

$$y = v_0 \sin\theta\, t - \frac{1}{2}gt^2$$

$$\sin 18^\circ \approx 0.31$$

$$y = 28(0.31)(0.34) - 4.9(0.34)^2$$

$$y \approx 2.95 - 0.57$$

$$y \approx 2.38\, m$$

The wall height is 2m the ball height is 2.38m so it shows that it clears the wall

2. How much does it bend;

Magnus force approximation:

$$F_M \approx \rho r^3 \omega v$$

$$F_M = 1.2(0.11)^3(8)(28)$$

$$(0.11)^3 \approx 0.00133$$

$$F_M \approx 1.2 \times 0.00133 \times 224$$

$$F_M \approx 0.36\,N$$

Lateral acceleration:

$$a_z = \frac{F_M}{m}$$

$$a_z = \frac{0.36}{0.43}$$

$$a_z \approx 0.84\,m/s^2$$

Time to goal:

$$t_{goal} = \frac{25}{28\cos 18^\circ}$$

$$t_{goal} \approx \frac{25}{26.6}$$

$$t_{goal} \approx 0.94\,s$$

Lateral displacement:

$$z = \frac{1}{2}a_z t^2$$

$$z = 0.5(0.84)(0.94)^2$$

$$z \approx 0.37\,m$$

The ball bends about 37cm sideways which is enough to curve around the wall

3. Why this is optimal;

We want to maximise:

$$f(\theta, v, \omega) = P_{goal}(1 - P_{keeper})$$

- If the angle is too small then it will hit the wall
- If the angle is too big then it would go over the bar
- If the spin is too small then it wouldn't bend around the wall
- If the spin is too big then it will lose power and would be an easy save for the goalkeeper

The optimal point balances:

CLEAR WALL + STAY UNDER BAR + BEND AROUND WALL

Ronaldo’s typical free kick;

- High velocity
- Moderate spin
- Lower angle (between 17-20°)

Which creates:

- Wall clearance
- Late dip
- Side bend
- Reduced keeper reaction time

Overall...

Using a real life example we can see that even a 2° chance to the angle or a tiny chance in spin would result in the ball hitting the wall or missing the goal

This shows free kicks are an application of projectile motion, fluid mechanics and optimisation theory

A mathematician can calculate the necessary criteria to score a free kick with extreme precision but practice is what truly determines whether you can or not

In conclusion, mathematics provides the model but skill provides the result