When a Positive Test Isn’t What It Seems: A Clinical Journey into Bayes’ Theorem

Introduction

I come to this not as a mathematician, but as a student of medicine. In clinical settings, numbers aren’t just calculations. They guide real decisions about real people. A test result, especially a ‘positive’ one, often feels definitive. But its meaning is rarely as clear as it seems. Understanding why requires looking at the math behind it.

The Emergency Room

For the purpose of this essay, consider yourself to be a clinician in the emergency room evaluating patients and making decisions based on the information available to you.

It is 2 a.m. and this your second night on the shift. The monsoon has you tired, and it has been a slow day. Slow days make you lethargic, but the patients continue to present and each case requires your prompt evaluation.

You spring up, energized as you hear a patient being brought in.

Miss Jane Headway, a 24-year-old university student, presents with a three-day history of high-grade fever, severe muscle pain, headache, and vomiting. She reports poor oral intake and increasing weakness [1]. Noticing that she is dehydrated, you order that she be started on an IV drip of Ringer’s lactate.

She mentions that several students in her hostel have had similar symptoms over the past few days.

You proceed to examine her. Her temperature is 39° C (normal: 36.1° - 37.2° C), pulse rate 98/min (normal: 60 - 100/min), respiratory rate 14 breaths per min (normal: 12 - 20 breaths per min), oxygen saturation 97% on room air (normal: 95 - 100%), and blood pressure 110/70 mmHg (normal: 120/80 mmHg). There is no rash or bleeding noted.

Given the clinical picture, local clustering of cases, and the ongoing seasonal trend, you suspect **Dengue fever** [1] and decide to perform a **Rapid Dengue NS1 antigen test** [2].

The result returns positive. At this point, how confident are you that the patient truly has dengue?

Understanding the numbers

Consider a group of 1000 patients who present with similar symptoms during a dengue outbreak. It would be reasonable to consider that 40% of them actually have dengue fever. This means, 400 patients have dengue and 600 don't.

Now consider the test. Suppose the Dengue NS1 antigen test correctly identifies the disease 90% of the time, but gives a false positive result in 10% of the patients who do not have the disease. (All tests are man-made and are imperfect, there is always room for error and false positives.)

Therefore, among the 400 patients who truly have dengue, 90%, i.e. 360 of the patients will test positive.

Among the 600 patients who do not have dengue, 10%, i.e. 60 patients will also test positive.

This gives a total of 420 positive results. 360 true positives and 60 false positives.

In this setting, if a patient tests positive, the probability that they truly have the disease is 360 out of 420. An approximate of 86%. It seems reasonable to conclude that the patient likely has Dengue fever. As the treating clinician, you would proceed with appropriate supportive management, close monitoring, and further evaluation for warning signs.

When the context changes

As you finish managing Jane, another patient is wheeled into the emergency room, and your attention shifts immediately. A middle-aged man is brought in. You walk towards him, ready to take his history.

48-year-old Mr. Simon Morris presents with sudden onset shortness of breath and a sharp right-sided chest pain that worsens with breathing. He mentions that his symptoms began a few hours ago. You notice a slight swelling in his left leg. When asked about it, he reports that it developed over the past two days [3]. There is no history of fever, cough, or recent illness.

On examination, his temperature is 36.5° C (normal: 36.1° - 37.2° C), pulse rate 112/min (normal: 60 -100/min), respiratory rate 24 breaths per min (normal: 12 - 20 breaths per min), oxygen saturation 92% on room air (normal: 95 - 100%), and blood pressure 122/78 mmHg (normal: 120/80 mmHg). You are concerned about the swelling in his left leg. On examination, it is mildly tender. Chest examination is otherwise unremarkable.

On further questioning, he reveals that he has been on prolonged bed rest for the past month following a recent lower limb fracture.

The combination of sudden breathlessness, chest pain, and unilateral leg swelling raises concern for a thromboembolic event. Being on prolonged bed rest indicates prolonged immobilization, increasing the risk of stasis of blood in his lower limb veins. This may have caused a thrombosis (blood clot) in his leg. Thrombi from the lower limb often travel from their point of origin and become lodged in the vasculature of the lungs. Given these features, you consider **Pulmonary embolism** [3] and decide to perform a **D-dimer test** [4] (helps determine if one has a blood clotting condition. It is not a confirmative diagnostic for pulmonary embolism).

The test comes back positive. Does this mean the patient has a pulmonary embolism?

Bringing in the numbers again

Once again, consider a group of 1000 patients presenting with symptoms that would require a D-dimer test.

Unlike the previous case, Pulmonary embolism is relatively uncommon. It would be reasonable to assume that about 10% of these patients actually have a pulmonary embolism. This would mean 100 patients have the disease and 900 don't.

Suppose the D-dimer tests positive correctly 90% of the time, but gives a false positive result in 30% of those who do not have a susceptibility to thrombus formation.

Therefore, among the 100 patients who truly have a pulmonary embolism, 90% i.e. 90 patients will test positive.

Among the 900 patients who do not have the disease, 30% i.e. 270 patients will also test positive.

This gives a total of 360 positive results. 90 true positives and 270 false positives.

In this setting, if a patient tests positive, the probability that they truly have a pulmonary embolism is 90 out of 360, i.e. only 25%. Despite a positive test and a concerning clinical picture, the diagnosis is far from certain.

Despite the concerning presentation and a positive D-dimer test, it would be inappropriate to make a definitive diagnosis of Pulmonary embolism at this stage. A positive D-dimer is not specific and can be seen in a variety of conditions. In this context, it only indicates that further evaluation in this direction is necessary.

As the treating clinician, the next step would be to proceed with confirmatory imaging to establish the diagnosis. Management decisions would depend on the results of these investigations and the overall clinical assessment.

What, then, explains this difference?

In both cases, the tests were similar in their accuracy. Both the patients had symptoms that justified investigation. Both received a positive result. And yet, the conclusions were entirely different.

The answer lies in Bayes' Theorem.
Bayes' Theorem describes how we update our beliefs in the light of new evidence. The probability of a condition, given a test result, depends not only on the accuracy of the test, but also on how likely the condition was to begin with [5]. Tests are flawed. They often detect things that don't exist and potentially flawed tests need to be treated accordingly. People often use test results without adjusting for test errors. Suppose you are searching for something as rare as 1 in a million. Even with a good test, it is likely that a positive result is really just a false positive on somebody in the 999,999 [5].

In the first case, the disease was common. The positive test added to an already high likelihood, making the diagnosis highly probable.

In the second case, the disease was relatively uncommon. Despite a positive result, the initial likelihood was low, and the presence of false positives meant that uncertainty remained significant.

Bayes' Theorem

To understand this mathematically, we begin by considering a simple question. What is the probability that a patient has a disease given that their test is positive? In other words, we are interested in how many of the positive results are truly meaningful.

This depends on two things:
1. How many patients actually have the disease to begin with, and how often does the test correctly identify them?
2. How often does the test give a positive result in those who do not have the disease?

Let's write down the formula [6].

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

A formal derivation of Bayes' theorem lies beyond the scope of this essay, but interested readers may refer to standard texts for a more detailed mathematical explanation [6].

Here,

$A =$ The event where the patient has the disease.

$B =$ The event where the test is positive.

Now,

$P(A|B)$ = Probability of A given B, i.e. probability that the patient has the disease GIVEN that the test is positive.

And,

$P(B|A)$ = Probability of B given A, i.e. probability that the test is positive IF the patient actually has the disease. (This just asks how good the test actually is at detecting the disease.)

Also,

$P(A)$ = Probability of the disease BEFORE doing the test. For a common disease like dengue during the season of spread, it is high. For a condition like pulmonary embolism, it is low.

And,

$P(B) =$ Probability of getting a positive test overall. Not just true positives. This includes false positives as well.

The formula shows that the probability of a disease after a positive test depends on three things:

1. How common the disease was to begin with?
2. How well does the test detect it? And,
3. How often does the test give positive results, including false positives?

Conclusion

In everyday medicine, decisions are rarely made in certainty. Every test result carries the possibility of error. False positives mislead and false negatives falsely reassure. The consequences of the decisions we make as clinicians are not theoretical. They directly affect patient care, making it essential to remain cautious, reflective, and aware of the limitations of the tools we use every day.

Medicine after all, does not offer perfect answers. There are no magical tests that can detect every disease with complete certainty. Every investigation is flawed in some way and only becomes a clue in a bigger search for the final diagnosis. Understanding these imperfections is as important as understanding the diseases themselves.

The cases discussed here along with the numbers used are simplified representations designed by me (as accurately as possible) to illustrate this principle. In reality, probabilities vary and clinical decisions are influenced by a wide range of factors. Clinical presentations are rarely this clear. As a primary student of medicine, not mathematics, this exploration is simply an attempt to bring the two fields together, to show how mathematical reasoning quietly underpins everyday clinical thinking.

Ultimately, Bayes' theorem is just one such principle among many that guide how we interpret evidence and make decisions under uncertainty. A test result does not give an answer, it reshapes what we already believe. Good medicine lies not in certainty, but in thoughtful judgement.

References

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