<u>Why Chess Players Only Learn Openings</u>

By Luc Didier

Chess is a board game invented in 6th Century India during the Gupta dynasty. It consists of a 8 by 8 chequered board made of light and dark squares. It consists of 2 teams, white and black. They each have the same amount and type of pieces as follows; 8 pawns, 2 rooks, 2 knights, 2 bishops, 1 queen and 1 king. They each would take turns of moving and playing the pieces with white always starting. The game would continue until one of the players have no legal moves to perform or are in check with no way to move the king. There are billions of possible games in chess and many impossible to play. A game of chess is usually around 40 moves before the game ends yet most chess players learn alot openings (a set of moves to begin the game with) for a game of chess and apply it to their games. But the question is why? In this essay, we will look at why chess players learn openings instead of midgame and endgame strategies. This will be broken down into 3 key parts: amount of unique games per turn, amount of unique possible checkmates in a game and then combining these 2 pieces of information to show why chess players only learn openings.

The first thing to define before we move anymore forward, what is a turn in chess? A turn consists of both players moving one of their pieces once after each other. It will always be white to start the turn and then followed by black. So one white piece move and one black piece move creates a turn. Games can of course end without a turn being complete since checkmate (A player's king is in check and can't move) or stalemate (The player moving a piece can't do a legal move but their king isn't in check) being delivered to black and them not moving. This would still count as a whole turn since it ends the game of chess.

The next thing to talk about is how the chess pieces move. The simplest pieces are the pawns, they can move one space forward and capture pieces (the action of going to a piece's space of the opposite colour and then removing them from the chess board) diagonally forward. On their first turn, they have the option to move 2 spaces forward instead. The rook (or castle) can move in any direction for any amount of spaces that isn't diagonally. They also capture pieces by landing on them on any piece they move onto. Then there's the bishop which can go in any direction but only diagonally and also if they land on a piece then they capture it. Then there's the knight (or horse) which moves 2 squares in any direction that isn't diagonal and then immediately moves one more square at a 90 degree angle to the end of the 2 squares. This piece is the only piece that can jump over pieces without capturing them and keep moving. Then there's the queen which can move like any piece except for the knight. Finally, there's the king which can't be moved into being captured and can only move one square in any direction around it at a time.

Now we have explained the terms and pieces, let's begin the first section.

1 - Finding the amount of unique games per turn

To start, let’s assume the game hasn’t begun yet. No moves have been done yet so therefore there are 0 unique games. On white’s move, they can move 10 different pieces, each with 2 different ways they can move. The 8 pawns in the front row or the 2 knights behind in the second row. This gives 20 possible games before black has even moved. When black moves, they also have 20 possible moves. To give all possible combinations from turn 1, we do 20 x 20 which gives 400 possible games already. This explodes very quickly to the point where at 40 games (the average chess game length) it is over 10^{120}.

To see where this number has come from, let's assume the chessboard but there is a coin every square, so there are 64 coins on 64 squares. Every turn, you flip a coin and put it back on the square starting in the bottom right corner and working your way through the chess board, only flipping each coin once. On the first turn, there are 2 possible board states. On the second, 4. The fifth would be 32 possible board layouts. The 10th would be at 1024. After the final coin flip, there would be 1.84×10^{19} possible states to 2 decimal places.This is a lot less than the 10^{120} but gives an idea on how the numbers blow up so quickly.

The equation for the coin flips is just 2^x. But how would you figure that out if you didn’t know that? So let’s solve it like it was a regular equation.

n	1	2	3	4	5	6	7	8	9	10
f(n)	2	4	8	16	32	64	128	256	512	1024

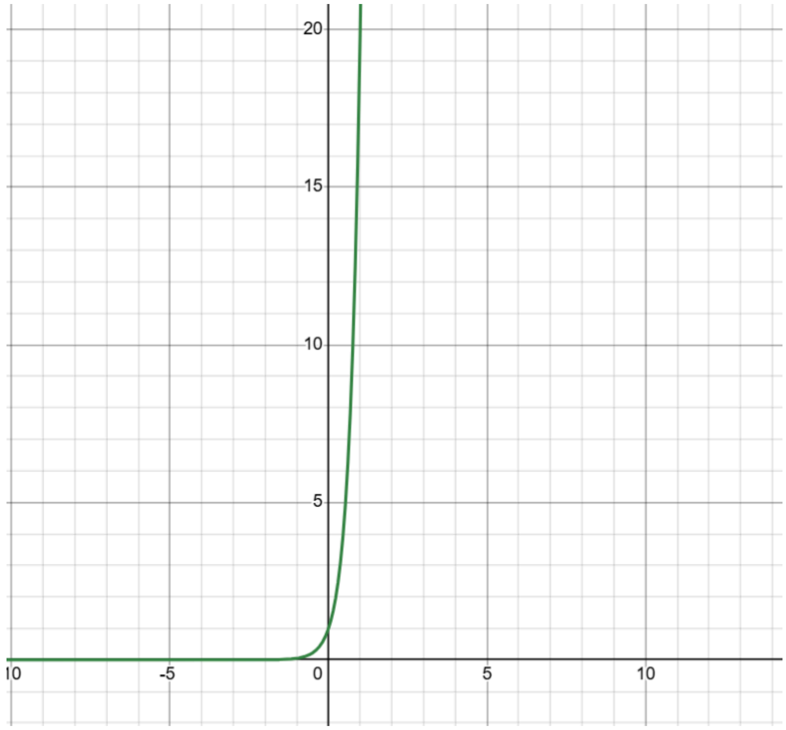
2 4 8 16 32 64 128 256 512

As you can see from the workings, it is the same equation. This would lead us to believe that it’s the exponent that changes and not the base of it. This leads us to see that it is the equation 2^x. Linking this same method to find the base of the amount of moves, we use the xth root to find what the base is raised. Interestingly, the equation results in the first base being 400 but the last one being 1000. So I searched for the equation on the internet and found it to be:

$$\text{Number of games} \approx \text{Average moves per position}^{2(\text{The number of turns})}$$

So thank you to Claude Shannon for the number, I couldn’t find where the equation came from so I assume the god of sequences gave it.

Now saying the average moves per position is 20, then the graph of it all will look like this:



This will relate to later on.

2 - Finding the amount of unique possible checkmates in a game

It is much harder to find out all the information for the amount of checkmates. However, what is known is that as the number of checkmates possible in a game decreases as the game continues. This means that once you reach a point, you can easily see a way to reach a checkmate in a chess game. This is because there are less pieces and less possible moves that can be done to end the game. For example, at the beginning of the game, it is possible to reach checkmate in every single way. As the game progresses, it eventually arrives at a point where checkmate can easily be seen as less can be reached and a clear path formed.

To begin, let's assume that there are 10,000 different possible checkmates at the beginning of a game. As each player moves their pieces, the amount decreases. So let's then assume that by the time it reaches the endgame it is at the endgame, there are 2 possible checkmates, one for each player. It would be a gradual decrease until around the midgame which creates a steep decrease and in the endgame would return to a gradual decrease. This would result in a reverse sigmoid curve.

The sigmoid curve function is as followed:

$$y = 12/(1 + e^{-x+e})$$

This produces the following curve:

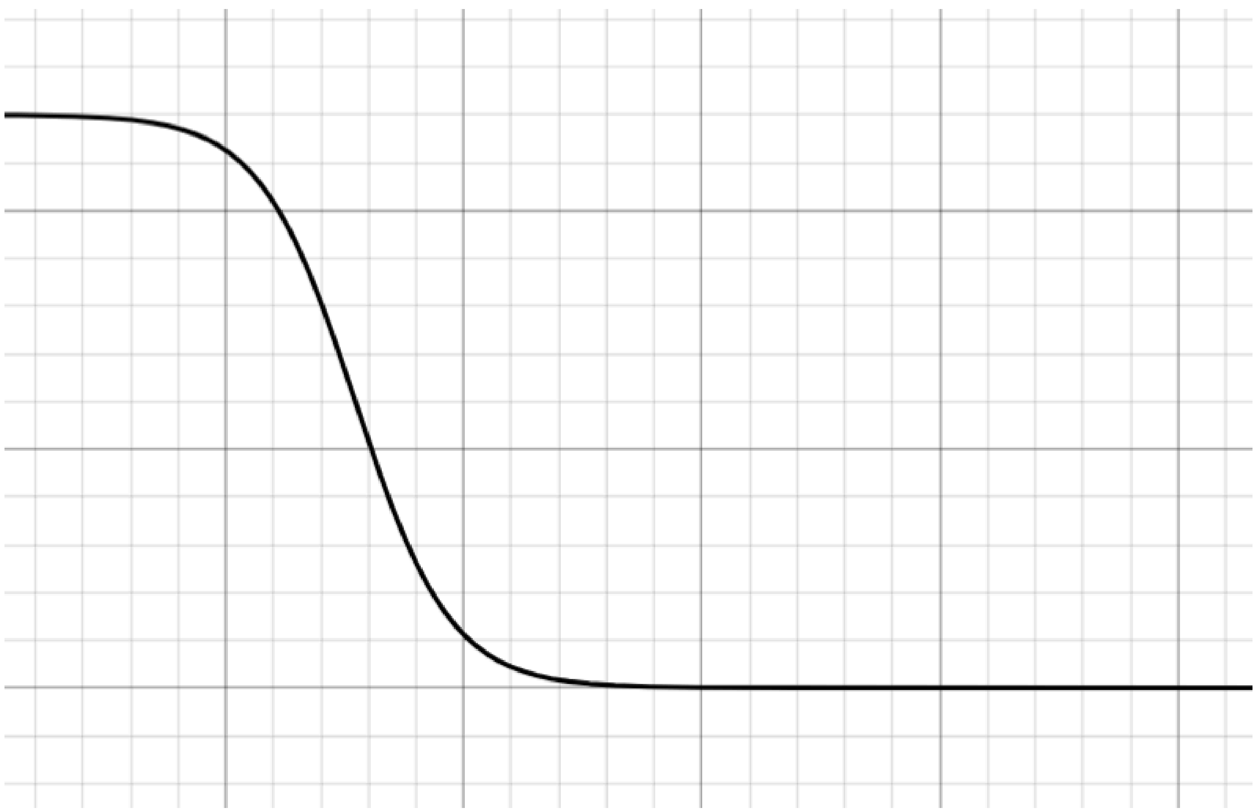
To inverse the function, many different signs were inverted or changed their value to result in inverting the numerator of the fraction. This inverses the S function (Sigmond function).

This results in the following S function:

$$-\ 12/(1\ +\ e^{-x+e})\ +\ 12$$

The + 12 on the end of the function is to adjust the function so that it results in being positive all the time and never negative.

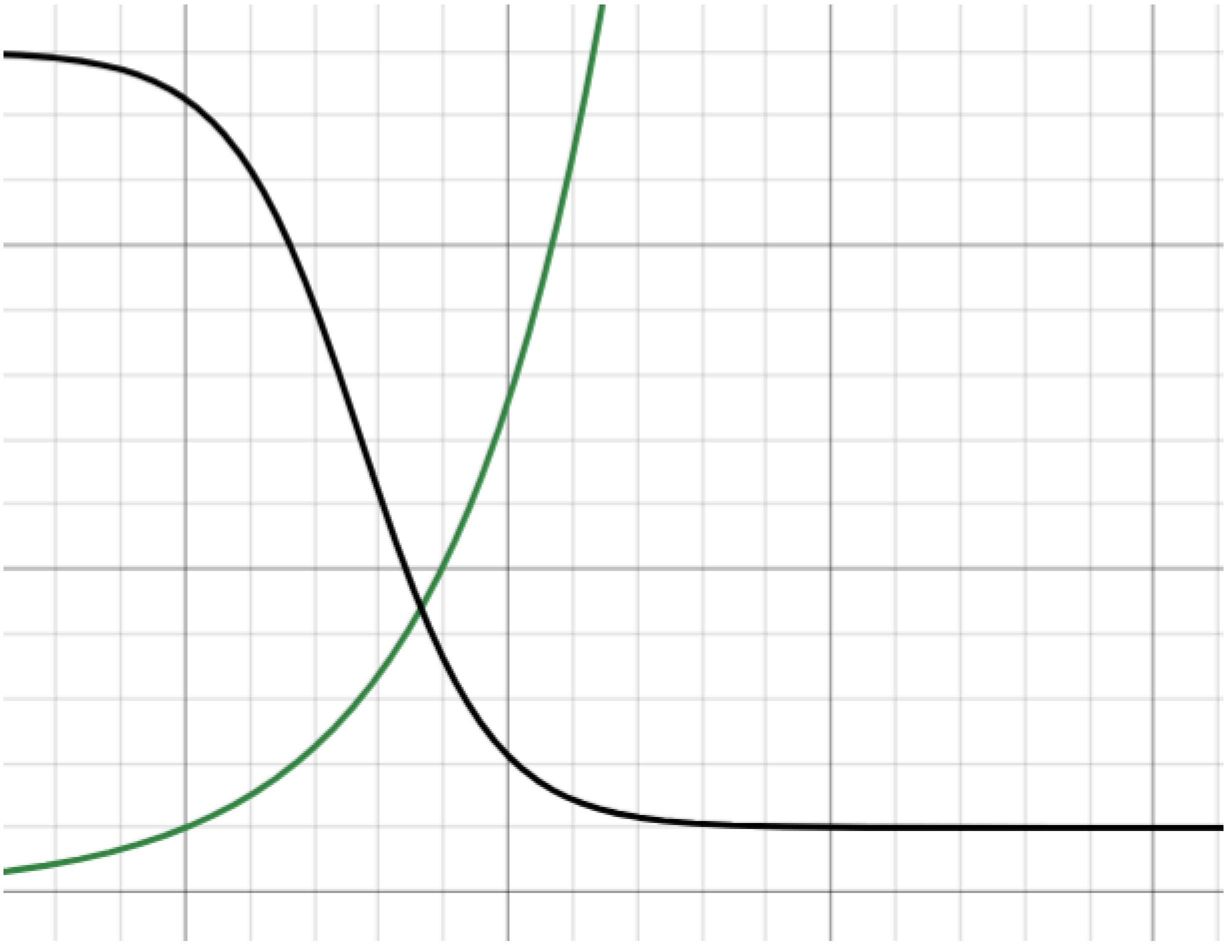
This results in the following graph to show the amount of checkmates during the game:



This is what was described earlier and shows the amount of checkmates possible during the game period as it approaches 1.

3 - Combining the information

Now for this final section, we will be looking at both graphs combined:

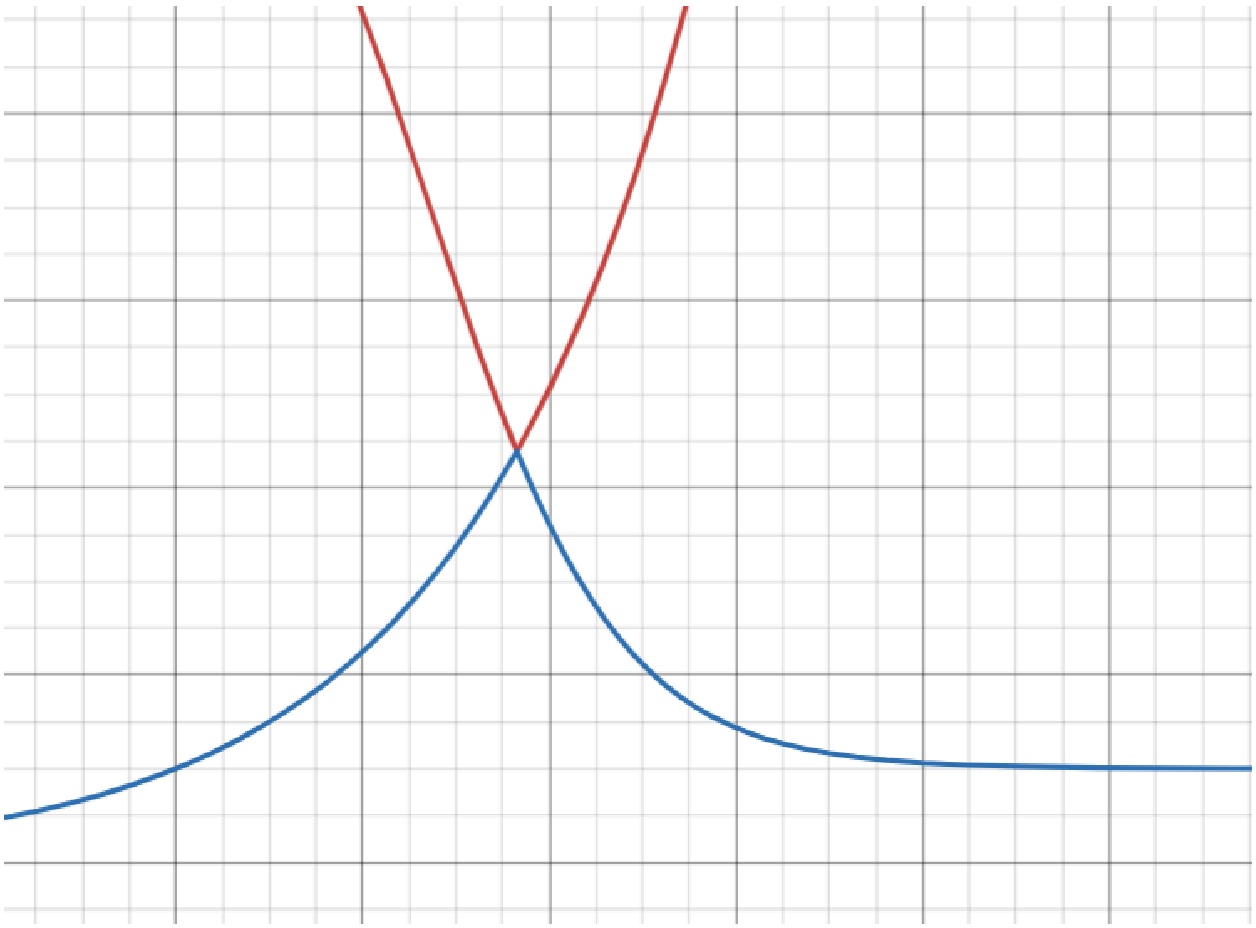


Green - Amount of unique possible games per turn, Black - Amount of unique possible Checkmates per turn

This graph will be vital to this section.

First, think about the human brain. It makes billions of calculations and functions every second but how does it prefer to store information? If you study information, which is easier; remembering the entire textbook or remembering a section of the book? The answer is obviously the section of the book since it has less information. This applies to all aspects of the human brain. It only really remembers the least and most important information. This is why memories have gaps and you don't remember everything for GCSEs.

This also applies to the combined functions. If we look back at the functions together, the smaller the number, the easier to compute. So what we do with the graph is remove the higher lines of the graph like so:



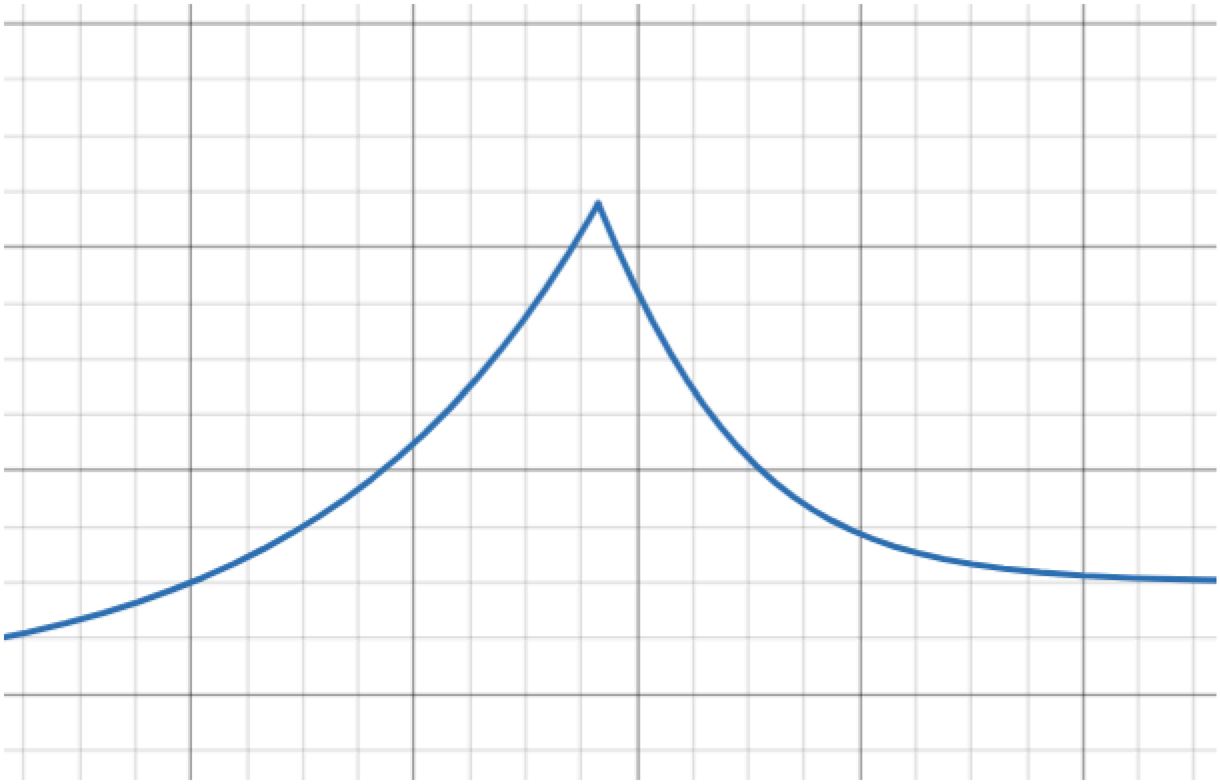
The lines with the greater numbers are highlighted red and the smaller numbers are highlighted blue to see what is wanting to be kept and removed. Spoiler: We want to remove the red. So we add limits to both functions. The limit for both is as follows:

$$y < \text{the point of intersection}$$

Which works out to be equal to exactly 4.39195. So the final limit of the functions are:

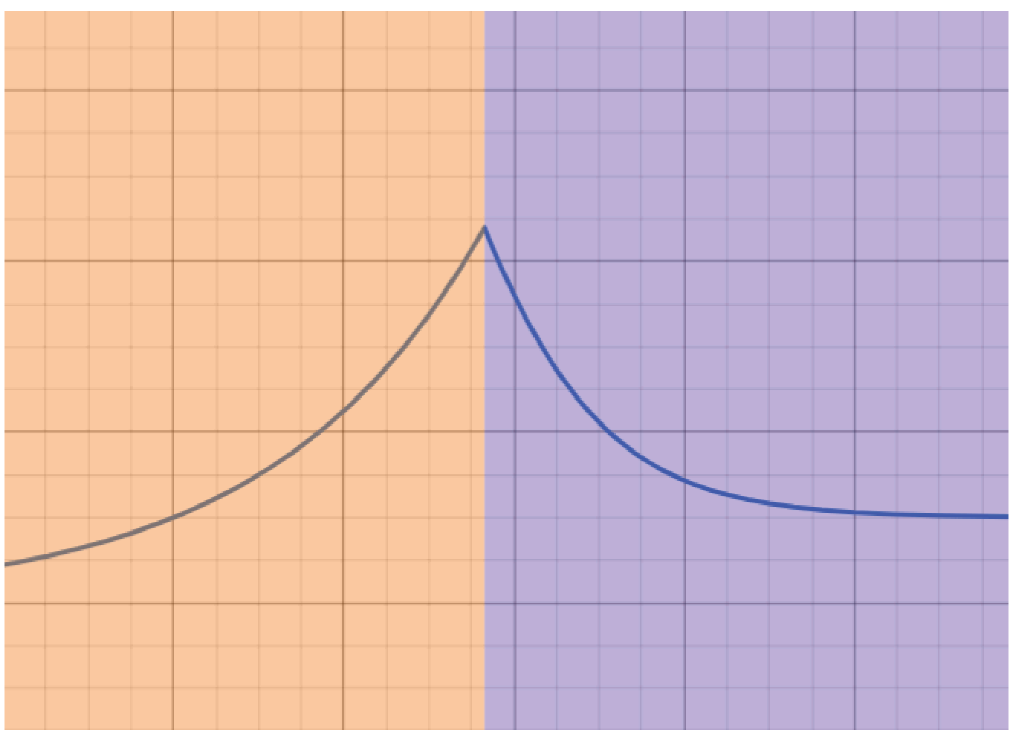
$$y < 4.39195$$

This results in the final graph of:



This just looks like a mountain now I guess.

If we look at each side of the graph now, it is 2 different sections of the game combined.



The orange section of the new graph shows the opening and amount of unique possible chess games and the purple section is the endgame and the amount of unique possible checkmates in a game.

These sections also correlate so that when the graph is going up, it is easier to learn since the amount of possible games is consistent. This means when the graph is going down, it is easier to read the endgame since the checkmates are easier to find. This is why chess players learn openings and never really closings or midgame strategies.

Final Words

I know that this may've not made sense or is confusing, well this is because this was my first ever maths essay and I wrote it in 2 days. A friend told me about this competition and I decided to take part after some debate with myself. I know I probably won't win but it was very fun to do this so thank you.

Sources:

https://en.wikipedia.org/wiki/Shannonnumber#citenote-2

https://artsandculture.google.com/story/a-game-of-thrones-how-chess-conquered-the-world-salar-jung-museum/fgUhNlxUQVZ2Kg?hl=en