Why does modelling stock markets as purely random fail?

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1 Introduction

Financial markets are often modelled as random systems, where price changes are treated as unpredictable fluctuations with some underlying trend. This has led to the development of Geometric Brownian Motion, one of the most widely used models of stock price movements. While this model captures the randomness observed in financial markets, real market behaviour suggests that it is too simplistic to fully describe price dynamics.

In this essay, I will examine the Geometric Brownian Motion model and its limitations in modelling stock markets, and explain how features such as memory, volatility clustering, and fractal behaviour provide a more realistic description of market behaviour.

2 Geometric Brownian Motion

2.1 Symmetric Random Walk

To understand Geometric Brownian motion we first need to learn about random walks. The simplest way to model randomness is through a symmetric random walk.

Consider repeatedly tossing a fair coin and recording the sequence of outcomes, H representing Heads and T representing tails. For example, a sequence would look like this HTHTT. To keep track of the outcomes, we assign a number to each toss, if we get Heads write +1, if we get tails write -1. Assign value of toss i to ω_i.

Now we get to the random walk part. Imagine a person walking down a straight path step by step and after each step you flip a coin, if it is Heads they step to the left and if it's Tails they step to the right. As they walk forward their sideways position changes based on the sequence of tosses. If we plot this path on a graph (horizontal axis being sideways position and vertical axis being the number of steps forward) we get a zigzag pattern, an example of this is shown in Figure 1.

Rotating this graph 90^o clockwise and taking 100 steps instead, we get Figure 2. The y-axis has M_n labelled, this represents the sideways displacement from our original point after n steps and mathematically is the sum of all our +1 steps to the left and -1 steps to the right.

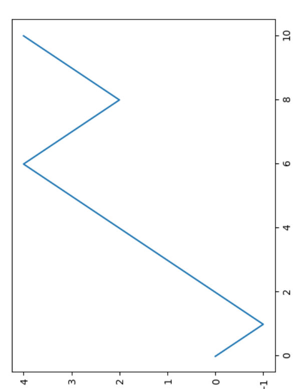

Figure 1: A path taken to get 10 steps forward

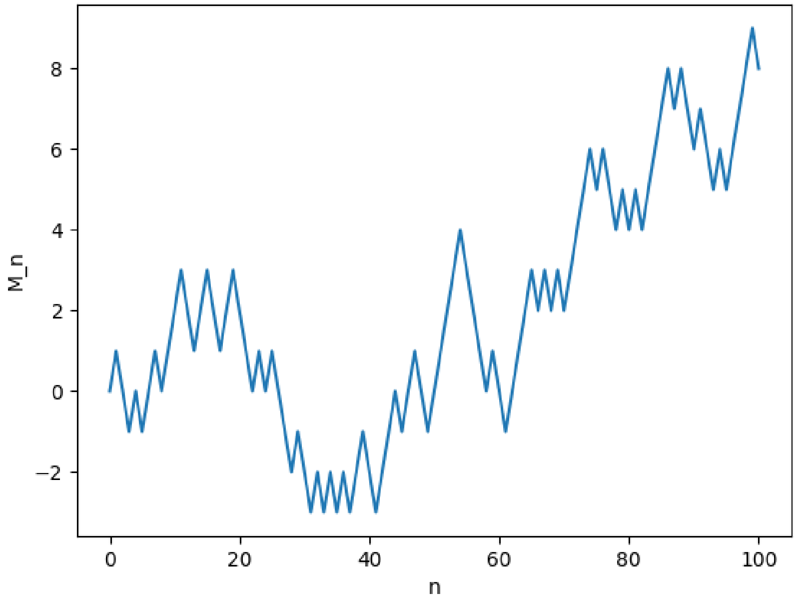

Figure 2: Symmetric random walk generated by matplotlib in python

Figure 2 begins to resemble the variation of the stock price over time.
This process can be written mathematically as

$$M_n = \underbrace{\sum_{i=1}^{n} X_i}_{\text{adding up all } X_i} = X_1 + X_2 + X_3 + \cdots + X_n$$

where

$$X_i = \begin{cases} +1 & \text{if } \omega_i = H \\ -1 & \text{if } \omega_i = T \end{cases}$$

where ω_i denotes the outcome of the i-th toss. [6]

The key feature of this process is that each toss is **independent** of the rest (the outcome of the previous tosses has no effect on the next toss) and that the **future is random**.

Note: Tossing a coin is not necessary for the symmetric random walk or Brownian motion, but it is a helpful way of thinking about whats going on.

2.2 Brownian Motion

Financial markets change continuously over time with price changes of various sizes, whereas the symmetric random walk is defined only for discrete steps with a fixed change in size (+1 or -1), this makes it too simple to model financial data. To improve this model we want to take smaller steps and more frequent steps to better approximate the continuously changing movement of stock markets.

To do this, we have to introduce a new variable, time. Now instead of just taking n steps overall we take n steps over a period of time, say from a time period from $t_0 = 0$ to $t = t_1$ we want to take n steps every unit time that passes. This means each interval $[0, 1], [1, 2], [2, 3]$... each have n steps of a random walk between them. We want to take n as very large as $n \to \infty$. However, our steps are still of size 1 so total displacement will grow without a bound. To prevent this we divide by $\sqrt{n}$ ensuring fluctuations remain bounded.

But why $\sqrt{n}$? First go back to $M_n = X_1 + X_2 + ... + X_n$, we want to find how far M_n typically is from its average. Find the average of M_n, each X has an equal probability to be +1 or -1 and so the

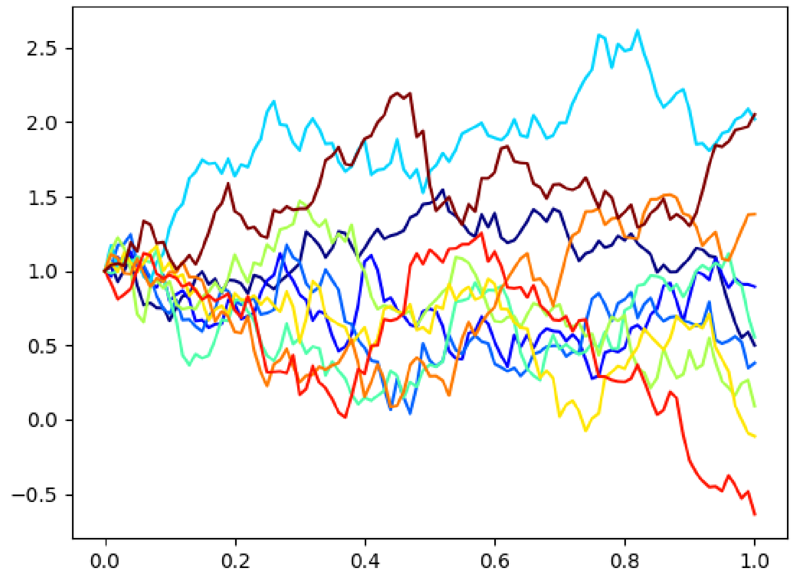

Figure 3: 10 Brownian Motion paths generated by Steven Golovkine [3]

average value of $\frac{(+1)+(-1)}{2} = 0$. Each of these have an average 0 then the average of the sum, M_n will have an average 0. Consider the maximum and minimum values of M_n (+n and -n respectively), so we say the Variance of M_n is n. Standard deviation is a measure of typical distance away from mean, which is defined as the square root of variance, so standard deviation $= \sqrt{n}$.

By scaling the random walk this way, we now have a process that behaves more smoothly over time, as n tends to infinity this converges to a process called Brownian motion. We define Brownian motion as

$$W(t) = \lim_{n \to \infty} \frac{1}{\sqrt{n}} M_{nt}$$

Here n is a large constant and t is the variable time, so as there are n steps every unit time there is nt total steps in the time period. A few examples of Brownian Motion is given in Figure 3. Brownian Motion, $W(t)$ is defined for all t in the time interval considered whereas the symmetric random walk is only defined for discrete times.

2.3 Geometric Brownian Motion

Although Brownian Motion gives a continuous model of randomness it still cannot describe financial markets properly, particularly because it allows values to go negative but stock prices are **always** positive. Additionally, this is **pure randomness** but to model stock prices we need to combine this randomness with a overall trend of the stock and changes being proportional to the current stock price.

This leads to Geometric Brownian Motion, the formula for this is

$$dS_t = \underbrace{\mu S_t dt}_{\text{general trend}} + \underbrace{\sigma S_t dW_t}_{\text{random fluctuations}}$$

where S_t is stock price at time t, μ is a constant of expected rate of return, σ is the volatility of the stock (how much the stock fluctuates over a period of time) , dt is a small time interval, dS_t is a small change in the stock price from time t to time $t+dt$, and dW_t is a small increment of Brownian Motion. [6] To understand this, consider a stock with price $S_t = \$100$, average rate of return $\mu = 5\% = 0.05$, $\sigma = 2\% = 0.02$. The first term $\mu S_t dt = 5dt$ gives a small predictable increase, $5dt$ represents a small growth. The second term $\sigma S_t dW_t = 2dW_t$ introduces uncertainty, this depends on size of σ and S_t and could be positive or negative. So at each moment the stock price increases slightly **on average** but also **unpredictably** due to randomness of Brownian motion. An example of Geometric Brownian motion is shown in Figure 4

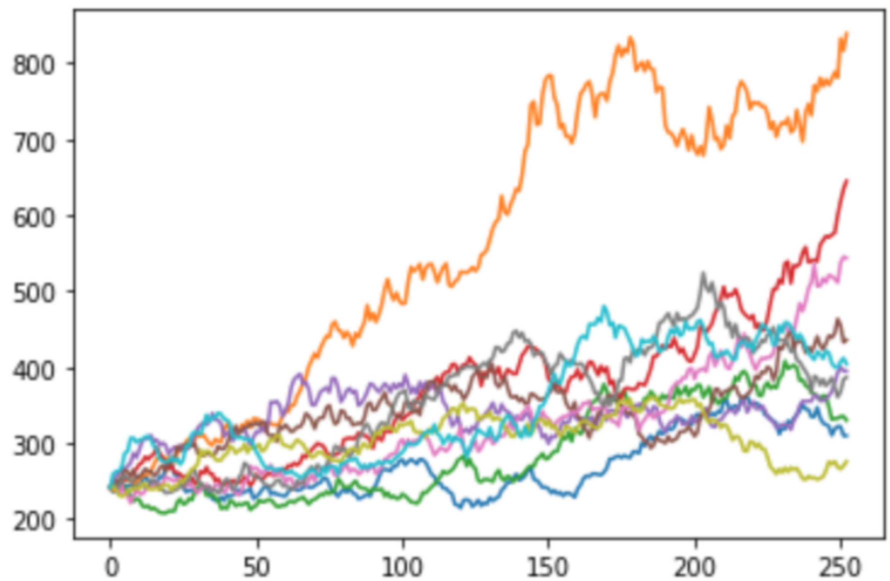

Figure 4: 10 Simulations of Geometric Brownian Motion modelling Microsoft's stock price by Jayati Walia[7]

3 Limitations of Geometric Brownian Motion

While Geometric Brownian motion is used wildly and provides a useful model of stock prices, its based on many assumptions which fail to entirely describe the behaviour of real financial markets.

The assumptions are:

- Volatility is constant
- Future price changes are independent of past movements
- No sudden crashes, price changes are gradual
- Stock returns are normally distributed (follow a bell curve)

3.1 Volatility Clustering

In Geometric Brownian Motion, σ the volatility of the stock is a constant which implies the price fluctuations are the same at every time, in reality there are periods of high and low volatility, an example shown in Figure 5. Periods of high volatility tends to be followed more large movements, while low volatility periods tend to continue with small movements, this is called volatility clustering [4]. For example, if there is a big price drop by \$10, some investors will sell their owned stocks anticipating further drops, this would increase supply and puts a downward pressure on price and causing price to fall further. However, other investors may buy more anticipating a future price rise, increasing demand therefore pushing the price back up. These opposing behaviours lead to large and rapid fluctuations in price. Conversely, when there is little buying or selling, price is likely to remain stable.

Volatility clustering suggests that markets are influenced by memory (past price movements influence future behaviour), particularly the recent past. While direction of future price is unpredictable still, the size of the price change is not entirely random. This is especially clear when looking at past financial crashes which are followed by large fluctuations. Therefore, both the assumptions that price is independent of the past and that volatility is constant fail to fully model financial markets.

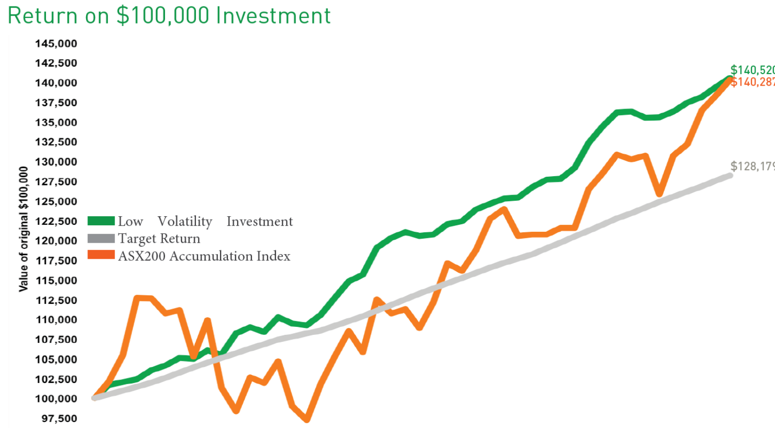

Figure 5: Green investment has volatility 0.04, orange investment has volatility 0.12 [8]

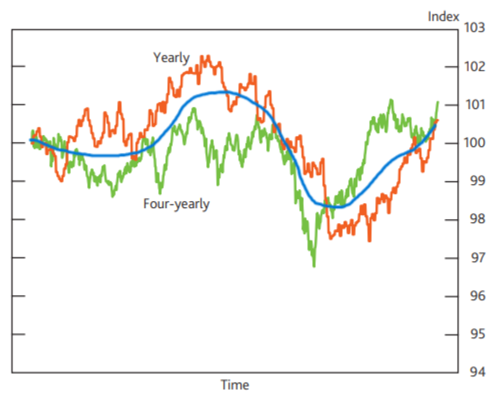

Figure 6: Annual (orange) vs four-yearly (green) observations of the Dow Jones industrials index (1988-97) [1], the general trend shape is in blue

3.2 Fractals in markets?

The existence of memory suggests that stock price movements follow a structure over time. We can describe this movement through fractals. But first of all, what are fractals?

There have been many definitions of what a fractal is but the most useful one in our case is: A fractal is an object that is self-similar when viewed at different scales. For example, if you take a branch of a tree, rotate and scale it up, it looks similar to the tree itself [2]. In a similar way, when stock price movements are examined over different periods, days, weeks, months, they display qualitatively similar patterns.

For example, in Figure 6 the yearly graph has been scaled according to the four-yearly graph to show the similarity in shape.

This behaviour contradicts the assumptions in modelling using Geometric Brownian motion as there seems to be an underlying structure and future prices are dependent on more than just S_t the current value but also on the past history of the stock. Therefore, fractal models provide a more suitable method for capturing the fractal behaviour of markets. This behaviour can be quantified by the Hurst exponent, H, which measures how strongly past price movements affect future prices using fractal analysis and is used in fractional Brownian Motion. Geometric Brownian motion uses the Hurst exponent (which is hidden) with $H = 0.5$, which is completely random. $H \in [0, 1]$, if $H > 0.5$ the process is persistent, so trends are more likely to continue. For example,if the price is increasing, it has a tendency to keep increasing. If $H < 0.5$ the stock is anti-persistent, trends are more likely to reverse, so an increase in price has a tendency to be followed by a decrease.[5]

This demonstrates that price movements are not purely random, but are dependent on the past fractal-like behaviour of the financial market.

4 Conclusion

Geometric Brownian Motion provides a simple and useful model for stock prices, combining randomness with an overall trend. However, its assumptions do not fully reflect real markets, which exhibit memory, volatility clustering, and fractal-like behaviour.

This shows that price movements are not purely random, but influenced by past behaviour and underlying structure. While the model remains widely used, it fails to capture several key characteristics of real markets.

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