<u>Why I haven’t cured cancer (yet)</u>

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Introduction

I believe there comes a point in every mathematician's journey when they have an epiphany- a revelation that mathematics is the profound language of the universe that all laws of nature link back to. Then the phase ends and they go back to complaining about maths homework. Either way, when my biology teacher suggested that mutations that cause cancer were "random", I simply was not having it. Surely that just means that there *is* a mathematical pattern we haven't had the competency to discover yet? Right? So, I did what anyone would do, I googled it. Up came a long list of technical biological concepts, which I was totally not buying. But also, this...

For a long time mathematicians were on the same boat as me, (except they were more on a yacht) and clung to this hope that if they kept brute forcing it, they would eventually come up with a complete, consistent system built from axioms, where every possible true statement had been proved. People *seriously* dedicated their lives to this, each one desperately trying to discover new proofs, until 1931 when Austrian mathematician (and logician) Kurt Gödel "broke" maths. Well, at least he broke a very ambitious dream that mathematicians had about it; Gödel proposed the idea that their proofs could ultimately mean nothing*, because every perfectly logical mathematical system was flawed- it must also consist of true unprovable statements that are not derivable from axioms. Essentially, there will always be some mathematical truth that lies beyond the reach of the system, which drastically impacts the rest of the system. This wreaked havoc in the mathematical community, and in this essay (after waffling a bit more about what Gödel actually did) I'll detail why. But first, let's talk about paradoxes.

*Ok, I'll admit for the purpose of this introduction this is a sumptuous exaggeration, and I would like to make it clear Gödel did *not* aim to tell mathematicians their work was useless. That would just be mean.

Paradoxes

An intuitive starting point when discussing Gödel's theorem is paradoxes. A notable verbal paradox is the liar paradox:
"This statement is false."

Let us assume this statement is true. Then, in fact, it is not false. So, the statement being "false" is false. But that would make the statement true. And so, you become stuck in this true, false, true, false loop. In literature and language there are many examples of these paradoxes, such as the Grelling- Nelson paradox (ironically formulated by *another* Kurt) ** and in philosophy perhaps the most notable is Russell's paradox (which we'll get into a few lines later). Paradoxes in language can arise when statements are self-referential. Basically, the paradox above works because it talks about itself. Another similar example- suppose a sign on an island reads "All statements on this island are false." If the statement is true, then, according to itself, it must be false. These paradoxes reveal an important concept about any system; if a system is allowed to refer to itself, it causes problems.

Anyway, as promised, let's consider the famous logician come philosopher, Bertrand Russell's paradox:

"Let *R* be the set of all sets that are not members of themselves. If *R* is not a member of itself, then its definition entails that it is a member of itself; yet, if it is a member of itself, then it is not a member of itself, since it is the set of all sets that are not members of themselves."

$$\text{Let } R = \{x \mid x \notin x\}\text{. Then } R \in R \iff R \notin R.$$

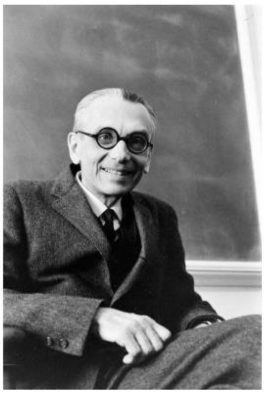
**A heterological adjective is one that does not describe itself- so is the adjective heterological itself heterological? Well, if it's not then surely it is heterological? And so on...

Wow. That's a mouthful, let's break it down. Russell considered the set of all sets that do not contain themselves. Does this set contain itself? If it does, then, by definition, it shouldn't. If it doesn't, then by definition it should. And there comes the contradiction. This was the first instance where mathematicians realised they weren't immune- paradoxes were not limited to language, they could sneak themselves into mathematics. To avoid this chaos, they dreamed of building a formal system: a set of axioms (basic assumptions) and rules of logic from which all mathematical truths could be derived, safely and consistently.

German mathematician David Hilbert proposed that, if we could formalize all of mathematics, then in principle we could prove every mathematical truth. He wanted this formal system to be:

1. Consistent- it could never produce contradictions.
2. Complete- every truth could be proven.
3. Decidable- there would be a procedure to determine if any statement was true or false.

It was a beautiful idea. But Gödel had other plans.



Kurt Gödel, moments before breaking mathematics:

<u>Gödel's incompleteness theorem</u>

Whilst Russell only proved the limitations of naive set theory, bringing light to its inconsistency, Gödel took it two steps further. He questioned whether any mathematical system, assuming it is complete and consistent, could be completely provable. So, how did he go about doing this?

Well, like we did, he started by looking at verbal paradoxes. As we've discussed, some paradoxes are self-referential statements. What Gödel figured out was that if you could translate mathematical statements into numbers, you could make numerical self-referential statements.

Constant sign	Gödel number	Usual Meaning
~	1	not
∨	2	or
⊃	3	if...then...
∃	4	there is an...
=	5	equals
0	6	zero
S	7	the successor of
(	8	punctuation mark
)	9	punctuation mark
,	10	punctuation mark
+	11	plus
×	12	times

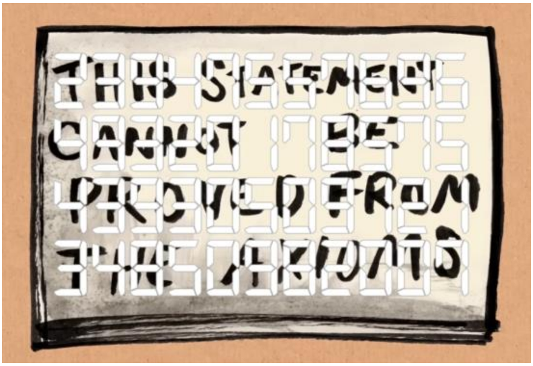
We now call this Gödel numbering (creative, I know) and it assigns a number to every mathematical symbol, formula, proof step and axiom in a system. True statements such as the Pythagorean theorem*** would have a code number and even false statements such as "17 is an even number" would have a code number.

***In a right- angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Suddenly, mathematics could talk about itself just like the liar paradox. Using this, Gödel created his own paradoxical statement that essentially said:

“This statement is not provable from axioms.”

with the statement itself obviously having its own unique Gödel number.



If the system could prove the statement using axioms, it would be proving something that says it cannot be proven- an obvious contradiction. If the system cannot prove the statement, it *is* true but unprovable- and an unprovable statement cannot be true. Unfortunately, maths is a lot less wishy-washy than linguistics- statements should be either true or false, not both!

So, let’s sort this out.

We’ve established the statement cannot be false, as it gets us stuck in that loop of contradiction (which clashes with the assumed consistency of mathematics). The only way to avoid this is by saying that the statement is true- *unprovably* true (from the axioms of mathematics). So, the only way to prove it true is to work outside of our perfectly logical system and add new axioms, such as the statement itself. Since the statement is true, it no longer adds inconsistency to the system. Except, adding the statement as an axiom means you can have another (almost identical) statement which can be proved from axioms. And this can go on infinitely with the only solution being to keep extending the system.

What Gödel was getting at was that no matter how many axioms you add to the system, there will always be something missing, which crushed mathematicians' dreams of one day having a 'complete' system like Hilbert suggested.

Okay... so why do we care?

Great question! Especially since you've made it through a thousand words of preface. Well, the truth is that Gödel's incompleteness theorem doesn't just sit in the abstract realm of numbers and paradoxes- it has real implications for mathematics itself. An example is the Paris-Harrington theorem, a strengthened version of a principle in combinatorics****. This theorem is interesting because it's true but it cannot be proven within the standard axioms of arithmetic (Peano arithmetic). Mathematicians can only see the theorem is true if they step outside the system, but within the system itself, it's unprovable- a direct illustration of Gödel's point that there are always truths beyond formal proof.

Suddenly, Gödel's theorem stretched further than his own weird little corner of maths and led mathematicians to question their entire life's work. What if they were hastily working away, trying to prove a theorem that was fundamentally unprovable (from the axioms of mathematics)? Uh oh.

This completely transformed the way mathematicians thought about their field.

****Combinatorics is the study of fine, discrete structures

Instead of assuming that every truth can eventually be proven, Gödel encourages us to accept that creativity and system expansion are an essential part of mathematical progress, and that the pursuit of mathematical proof (kind of like the pursuit of any truth, to be honest) opens up an infinite horizon.

Okay... so that's why mathematicians care, but why should *I* care?

Well, unfortunately I don't know you personally, but what I can do is explain why this links back to my original question.

Just as Gödel showed that even the most perfectly logical mathematical system can never capture every truth, cells in our bodies also operate under inherent limits and unpredictability. Take DNA, a blueprint for life. But this blueprint isn't fixed- in fact, it can mutate. A mutation is a change in the DNA sequence of an organism that can result from errors in DNA replication during cell division, exposure to mutagens, or a viral infection. Most mutations are harmless, some are even beneficial, and others can lead to diseases like cancer. The interesting parallel to Gödel's incompleteness is that, even if we know all the rules of genetics and molecular biology, we cannot predict every possible mutation or its consequences with absolute certainty. There will always be outcomes that are true in nature but unprovable in our models.

This unpredictability also has consequences for medicine, since cancer arises from the abnormal growth and division of cells due to mutations in a cell's genome. Even with the best understanding of cellular machinery, it's impossible to predict every mutation pathway a cell might take- just like the unprovable truths in mathematics. Researchers can deduce likely patterns and probabilities, but complete prediction is inherently impossible. This mirrors Gödel's insight that no formal system can ever fully capture all truths: in biology (at least in this example), the ‘system’ is the genome, and the ‘truths’ are the possible outcomes of mutations.

Although this demolished my initial hope of discovering the cure to cancer via a mathematical pattern, this isn’t actually a bad thing for science. Just like in maths, acceptance is the best way to go. Instead of desperately trying to anticipate each and every cell mutation that could cause cancer, it’s a lot better to design flexible gene therapies and adaptive cancer treatments that allow for holes in the system.

Conclusion

So, to sum up, Gödel proved no matter how far we venture through mathematics, biology or any other subject, nothing we do will ever be enough. Well, not exactly. Gödel’s deeper insight was more that we shouldn’t create these limits of mathematics just to ease our minds with something predictable and familiar. Instead, we should accept that there isn’t a pattern or proof for everything (like cancer), and the world is a lot deeper than just maths. Yes, unfortunately the world doesn’t simply revolve around mathematicians.

Don’t worry though guys, I promise I’ll keep on thinking of new ideas.

References so I don't get sued:

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