Why the Casino Always Wins: The Mathematics of Expected Value

Bright lights, spinning wheels, and the sound of chips stacking up high. Casino floors are great at making you believe that you're just one turn away from a life changing score. Every player believes that this might finally be the time where they win big and might be the lucky one. Yet, despite millions of random outcomes every day, casinos make consistent profits year after year. So it makes us ask the question, if gambling is truly random, then how come casinos always win?

The answer to this question does not lie in luck, however in mathematics and specifically in probability, expected value and variance. What appears chaotic in the short term becomes predictable in the long term. Ever notice there are no clocks or windows in a casino? That's not an accident. The longer they keep you in that loop of playing, the faster the long-term math catches up to you. They provide the free drinks and the flashy sounds to keep you distracted from the fact that you're essentially fighting a losing battle. By analysing a simple roulette bet, we can see how statistics guarantee the casino a profit.

Probability in Roulette:

European roulette contains 37 numbered pockets; they range from 0-36. A player can place a bet on a single number or on a colour.

For a single number bet: The probability of winning is:

P(win) = $\frac{1}{37} = 0.027$

For a colour bet: There are 18 red, 18 black and 1 green: Assuming a player bets on red:

P(win) = $\frac{18}{37} = 0.486$

Probability of losing:

P(lose) = 1 - P(win)

For a single number: $\frac{36}{37}$

For a colour: $\frac{19}{37}$

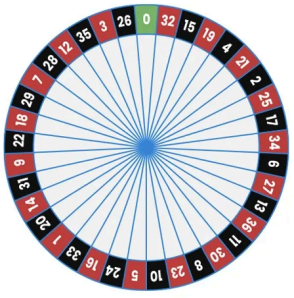

Figure 1: A European roulette wheel (0-36), showing red, black and zero

The probabilities that I have given already hint at why the casino is favoured: in both cases, losing is slightly more likely than winning.

Expected Value and Random Variables

Expected value (EV) represents the average outcome over time. Let's define random variable X as profit from one bet.

Single number bet:

X = +35 with probability of $\frac{1}{37}$

X= -1 with probability of $\frac{36}{37}$

Expected value:

$E(X) = 35(\frac{1}{37}) + (-1)(\frac{36}{37})$

$= -\frac{1}{37} = -0.027$

This means that a player loses on average 2.7p per £1 bet.

Colour bet:

X = +1 with probability of $\frac{18}{37}$

X = -1 with probability of $\frac{19}{37}$

$E(X) = 1(\frac{18}{37}) + (-1)(\frac{19}{37})$

$E(X) = -\frac{1}{37} = -0.027$

Even though the chance of winning is nearly 50%, the casino still wins over time because of the zero.

Bet type	Probability of win	Probability of lose	Payout	EV (£)
Single number	$\frac{1}{37}$	$\frac{36}{37}$	35	-0.027
Colour	$\frac{18}{37}$	$\frac{19}{37}$	1	-0.027

Figure 2: Expected value for different roulette bets

A crucial factor in the casino's advantage is the presence of the zero. If the roulette wheel only contained numbers 1-36, a single number bet paying 35 to 1 would be completely fair, and a colour bet would also have an expected value of zero. The inclusion of the zero slightly reduces the player's probability of winning and ensures a negative expected value of -0.027 for both bet types. This tiny mathematical difference is the foundation of the house edge. Over millions of bets, this tiny disadvantage accumulates into a predictable and substantial profit for the casino, demonstrating that the casino's success is a result of careful statistical design rather than luck.

Variance: Short Term Outcomes

Expected value tells us the average, but variance tells us why players sometimes win big.

Formulae:

$\text{Var}(X) = E(X) - [E(X)]^2$

Single number bet variance:

$X^2 = 35^2 = 1225$ win
$X^2 = (-1)^2 = 1$ lose

$E(X^2) = 1225(\frac{1}{37}) + 1(\frac{36}{37}) = 34.08$

$\text{Var}(X) = 34.08 - (-0.027)^2 = 34.08$

Colour bet variance:

$X^2 = 1$ win
$X^2 = 1$ lose

$E(X^2) = 1(\frac{18}{37}) + 1(\frac{19}{37}) = 1$
$\text{Var}(X) = 1 - (-0.027)^2 = 0.999$

Observation: Single number bets have much higher variance, big wins are rare but huge. Colour bets are smaller, steadier wins, bet EV is still negative.

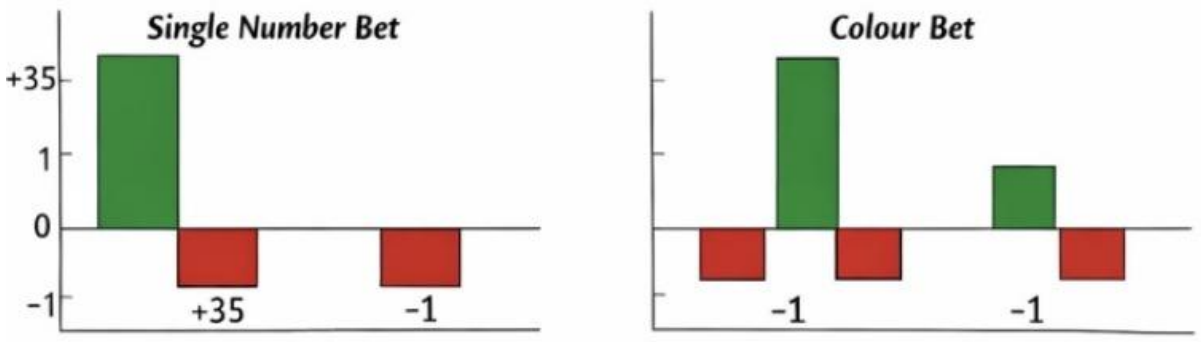

Figure 3: Variance shows why single number bets have large short term swings

Multiple Bets: Binomial Modelling:

We can model multiple bets using a binomial distribution. Let X~ Bin(n,p) be the number of wins in n bets.

We have 100 single number bets:

$E(X) = np = 100(\frac{1}{37}) = 2.7$ wins

Expected profit:

2.7(35) - 1(97.3) = -2.8

We have 100 colour bets:

$E(X) = 100(\frac{18}{37}) = 48.6$ wins

48.6(1) - 1(51.4) = -2.8

Even with different types of bets, the casino wins the same average percentage due to the zero.

Figure 4: Binomial distribution predicts expected wins in multiple bets

Law of Large Numbers

The law of large numbers ensures that as the number of bets increases, the average outcome approaches expected value:

$\frac{S_n}{n} \rightarrow$ E(X) as n $\rightarrow \infty$

Where S_n is total profit after n bets.

When it comes to one player, the outcome seems random. However, when millions of bets are placed in places like Las Vegas, then the average losses converge to 2.7%.

For Example:
10,000,000 (1 bets) (2.7%) = 270,000

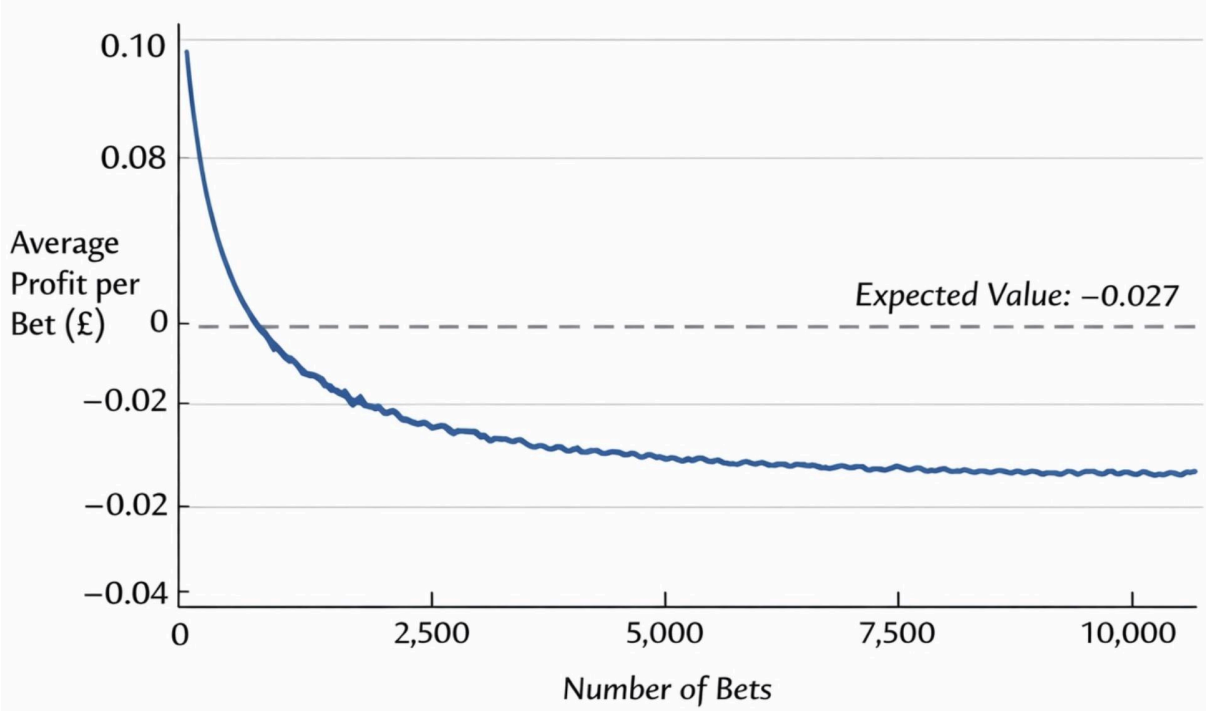

Figure 5: Average loss per bet converges to expected value

Conclusion:

Roulette might look like a game of pure luck. Short term outcomes vary due to variance, and players may win occasionally. However, mathematics shows that the casino's profit is inevitable:
Probability explains how likely each outcome is.
Expected value guarantees the player loses slightly on every bet.
Variance creates a short term excitement and occasional winners.
Binomial distribution shows expected results over multiple bets.
The law of large numbers ensures the casino always profits in the long run.

Whether betting on a single number or a colour, the expected loss is the same. Casinos do not rely on luck or cheating, they rely on mathematics. And in the long run, mathematics always wins.