Why unlikely events are more likely than we think?

Introduction

It is the start of the summer holidays and you have just laid down on the soft Maldives sand when out of the corner of your eye you notice that one really annoying coworker who you'd hoped never to see again. Coincidence? I think not. In fact, what we call a ‘coincidence’ is very often the result of predictable mathematical principles combined with flawed human intuition.

Why coincidences feel so surprising

The thing is, I can almost guarantee that most of your holidays have not unfolded in that way. The unlikely events USUALLY don’t actually happen. But when it comes to your Friday night dinner party with friends, those aren’t very interesting topics of conversation. Our biased brains tend to remember all the times the shocking coincidences did happen, and just completely overlook all the times they didn’t. Additionally, us humans are extremely bad at understanding very small and very big numbers. Something ‘unlikely’ to happen is not impossible - as much as it may feel like it. We intuitively underestimate how often ‘chances’ occur and hence our perception of probability is not based on calculation, but on selective memory and pattern-seeking.

The birthday paradox

One of the clearest demonstrations of how deeply our intuition misjudges probability is the Birthday paradox. You walk into a room with 23 people and are told that there is a 50% chance that two of you share a birthday. Absurd, maybe. But true, definitely. Why? After all, there are 365 possible birthdays, so how could such a small group produce a match so easily? The mistake lies in how we are thinking about the problem. We tend to compare one person to the rest - in which case indeed the probability of person A sharing a birthday with one of the 23 other people in the room is extremely low. However, what about person B? And person C? The number of distinct pairs is given by $n(n - 1) / 2$. For 23 people, this means a total of 253 separate pairings - 253 chances for a shared birthday to occur. And this number does not increase linearly but quadratically, so if your best friend were to then walk into the room, that would dramatically increase the number of possible pairs. The probability therefore accelerates far more quickly than our brains may expect. What appears to be an extraordinary coincidence is, in fact, a predictable consequence of combinatorial growth: not because any single match is likely, but because the sheer number of opportunities makes one almost inevitable.

Why “unlikely” events become likely

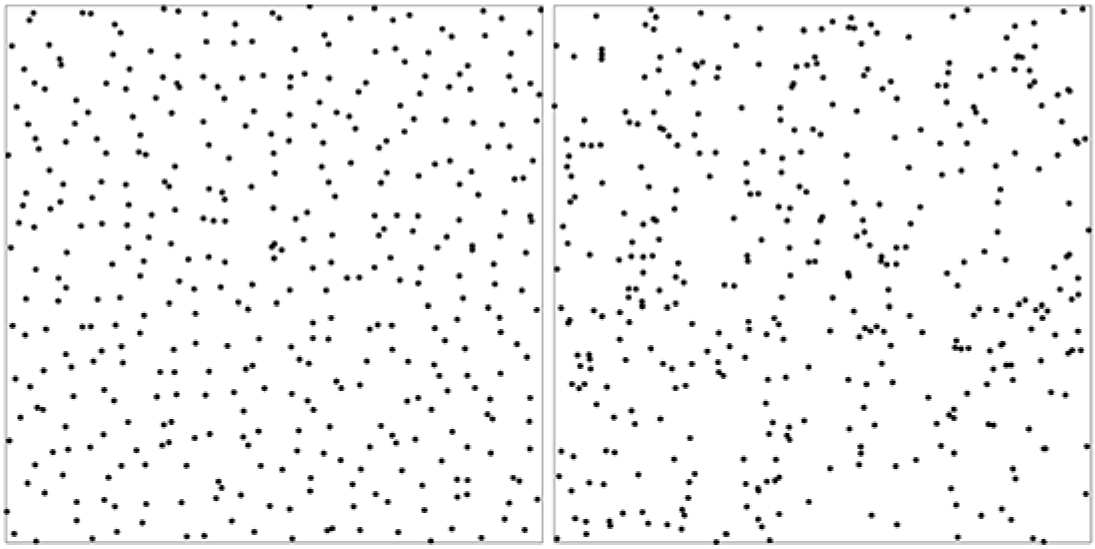
The logic behind the birthday paradox extends far beyond shared birthdays, pointing to a more general principle known as the Law of truly large numbers. This principle states that with a sufficiently large number of trials, even highly unlikely events become almost certain to occur. This is one of the features of cumulative probability: when there is a single opportunity or trial for an event to occur, the probability of the event occurring is extremely small. However, increasing the number of trials steadily increases the overall probability that the event will happen at least once. In other words, probability stacks up. An event with a one-in-a-million chance may seem impossible, but if there are millions or even billions of opportunities for it to occur, the odds shift. Take as an example everyday life, where we are constantly exposed to a vast number of opportunities for chance encounters. Under these conditions, coincidences are not exceptional, they are inevitable. What appears rare when viewed in isolation becomes almost guaranteed when considered across a sufficiently large scale.

Monty hall

If the birthday paradox demonstrates how we underestimate the number of opportunities for coincidence, the Monty Hall problem reveals a deeper flaw in how we update probabilities. You are invited onto a game show and are told to select one of three doors (behind which two are empty and one contains a prize). You select a door, and the presenter then opens one of the two remaining doors showing you that door is empty. Do you change your guess, or keep it? Although it appears that the odds are now evenly split, making the decision irrelevant, this intuition is in fact incorrect. Your initial choice had only a one in three chance of being correct, meaning the prize had a two in three chance of being behind the two other doors. When the host eliminated one of the two remaining options, that entire probability transferred to the remaining unopened door, making the switch the optimal strategy.

Randomness and patterns

While the Monty Hall problem exposes flaws in how we update probabilities, our misunderstanding extends further to randomness itself. Which of these two images appear to be the most random?



In reality, the image on the right represents true randomness, while the left shows a uniform distribution. People expect randomness to appear even, but the reality is far from that. Randomness creates clusters and streaks, and as a result, patterns are not evidence against randomness but rather a consequence of it.

Gambler’s fallacy

A further illustration of our misunderstanding of randomness is the gambler’s fallacy, the belief that past events can influence future probabilities in independent systems. For example, you are flipping a coin and heads appear 5 times. It may feel as if tails is now ‘due’ to appear, but that is entirely wrong. The probability of either outcome remains 50/50. This is again because we expect randomness to be evenly distributed even though so often it isn’t. What appears to be a meaningful pattern is often nothing more than the inevitable variability of random processes.

Conclusion

So, was that encounter on a distant beach really a coincidence? Perhaps, but not in the way it first appeared. As the birthday paradox reveals, the number of opportunities for unlikely events grows far faster than we expect and as the Law of truly large numbers shows, even the rarest events become inevitable when given enough chances to occur. Meanwhile, the Monty Hall problem and gambler’s fallacy demonstrate that our brains struggle not only with scale, but with the very idea of probability itself.

What unites these ideas is not that the world is full of mysterious coincidences, but that our perception of chance is fundamentally flawed. We expect randomness to behave neatly, probabilities to reset, and rare events to remain rare, but maths tells a different story. In reality, what we call a coincidence is often nothing more than the natural outcome of systems far larger and more complex than our brains can grasp.

Perhaps, then, the real surprise is not that unlikely events happen, but that we ever believed they wouldn’t.