GENERAL BACCALAUREATE

SPECIALTY SUBJECT EXAMINATION

SESSION 2024

MATHEMATICS

Day 2

Duration of the test: **4 hours**

The use of a calculator with exam mode active is permitted.

The use of a "college-type" memoryless calculator is permitted.

As soon as you receive this document, make sure it is complete.
This topic comprises 6 pages numbered from 1 to 6.

The candidate must complete the four exercises provided.

The candidate is encouraged to include on the copy any trace of research, even incomplete or unsuccessful, that he or she has carried out.

The quality of writing, clarity, and precision of reasoning will be taken into account in the assessment of the paper. Traces of research, even incomplete or unsuccessful, will be valued.

Exercise 1 (5 points)

The headmistress of a school wishes to conduct a study among students who have taken the final exam, to analyze how they believe they succeeded in this exam.

For this study, students were asked individually after the exam to answer the question: "Do you think you passed the exam?". Only "yes" or "no" answers were possible, and it was observed that 91.7% of the students surveyed answered "yes".

Following the publication of the exam results, it was discovered that:

- 65% of the students who failed answered "no";
- 98% of successful students answered "yes".

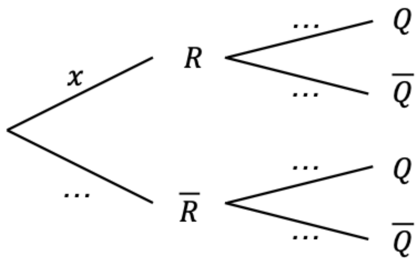
A student who has taken the exam is chosen at random.

On note R the event "the student passed the exam" and Q the event "the student answered "yes" to the question".

For an event A any, we note P(A) its probability and P(A-bar) the opposite.

Throughout the exercise, probabilities are rounded to the nearest whole number, if necessary to 3 decimal places.

1. Specify the probability values P(Q) and $P_{\bar{R}}(\bar{Q})$.



2. On note x the probability that the student interviewed passed the exam.
a. Copy and complete the weighted tree diagram opposite.
b. Show that $x = 0{,}9$.

3. The student interviewed answered "yes" to the question. What is the probability that he passed the exam?

4. The grade obtained by a randomly selected student is an integer between 0 and 20. It is assumed to be modeled by a random variable N which follows the binomial distribution with parameters (20; 0.615). The director wishes to award a prize to the students who achieved the best results.
At what grade level should she award the prizes so that 65% of students are rewarded?

5. Ten students are interviewed at random.

Random variables $\mathrm{N}_1, \mathrm{N}_2, \dots, \mathrm{N}_{10}$ model the grade out of 20 obtained on the exam by each of them. We assume that these variables are independent and follow the same binomial distribution with parameters (20; 0.615).

Either S the variable defined by $\mathrm{S} = \mathrm{N}_1 + \mathrm{N}_2 + \cdots + \mathrm{N}_{10}$.
Calculate the expected value $\mathrm{E}(\mathrm{S})$ and the variance $\mathrm{V}(\mathrm{S})$ of the random variable S.

6. We consider the random variable $\mathrm{M} = \mathrm{S}/10$.

a. What does this random variable model M in the context of the exercise?

b. Justify that $\mathrm{E}(\mathrm{M}) = 12{,}3$ and $\mathrm{V}(\mathrm{M}) = 0{,}47355$.

c. Using Bienaymé-Chebyshev inequality, justify the statement below.

"The probability that the average grade of ten randomly selected students is strictly between 10.3 and 14.3 is at least 80%."

Exercise 2 (5 points)

Parts A and B are independent.

Alain owns a swimming pool which contains $50\ \mathrm{m}^3$ of water. It is worth remembering that $1\mathrm{m}^3 = 1000\ \mathrm{L}$. To disinfect the water, he must add chlorine.

The chlorine level in the water, expressed in $\mathrm{mg.L^{-1}}$ Chlorine concentration (CVC) is defined as the mass of chlorine per unit volume of water. Pool professionals recommend a chlorine level between1 and $3\ \mathrm{mg.L^{-1}}$.

Under the influence of the surrounding environment, particularly ultraviolet radiation, chlorine decomposes and gradually disappears.

On certain days, at a fixed time, Alain takes measurements with a device that allows for precision at $0{,}01\ \mathrm{mg.L^{-1}}$ On Wednesday, June 19, he measured a chlorine level of $0{,}70\ \mathrm{mg.L^{-1}}$.

Part A: Study of a discrete model.

To maintain the chlorine level in his pool, Alain decided, starting Thursday, June 20th, to add a quantity of $15\ \mathrm{g}$ of chlorine. It is assumed that this chlorine mixes uniformly in the pool water.

1. Explain why this addition of chlorine increases the level by 0.3$\mathrm{mg.L^{-1}}$.

2. For any natural number n, on note V_n the chlorine level, in $\mathrm{mg.L^{-1}}$, obtained with this new protocol n days after Wednesday, June 19th. Thus $V_0 = 0{,}7$.
We admit that for any natural number n, $V_{n+1} = 0{,}92V_n + 0{,}3$.

a. Show by induction that for all natural numbers n, $V_n \leq V_{n+1} \leq 4$.

b. Show that the following(V_n) is convergent and calculate its limit.

3. In the long term, will the chlorine level comply with the recommendations of pool professionals? Justify your answer.

4. Reproduce and complete the algorithm opposite, written in Python, so that the function`chlorine_alert` returns, when it exists, the smallest integer n such as $V_n > s$.

5. What value do we get when we enter the instruction `chlorine_alert(3)`? Interpret this result in the context of the exercise.

```
def alert_chlorine(s) : n=0
v=0.7
while .................... :
n= ...........
in= ...........
return n
```

Part B: Study of a continuous model.

Alain decides to call in a specialist consulting firm. This firm uses a continuous model to describe the chlorine level in the pool.

In this model, for a duration x (in days elapsed since Wednesday, June 19th), $f(x)$ represents the chlorine level, in $\mathrm{mg.L^{-1}}$, in the swimming pool.

We admit that the function f is a solution to the differential equation $(E): y' = -0{,}08y + q/50$ or q is the amount of chlorine, in grams, added to the pool each day.

1. Justify that the function f is of the form $f(x) = Ce^{-0{,}08x} + q/4$ or C is a real constant.

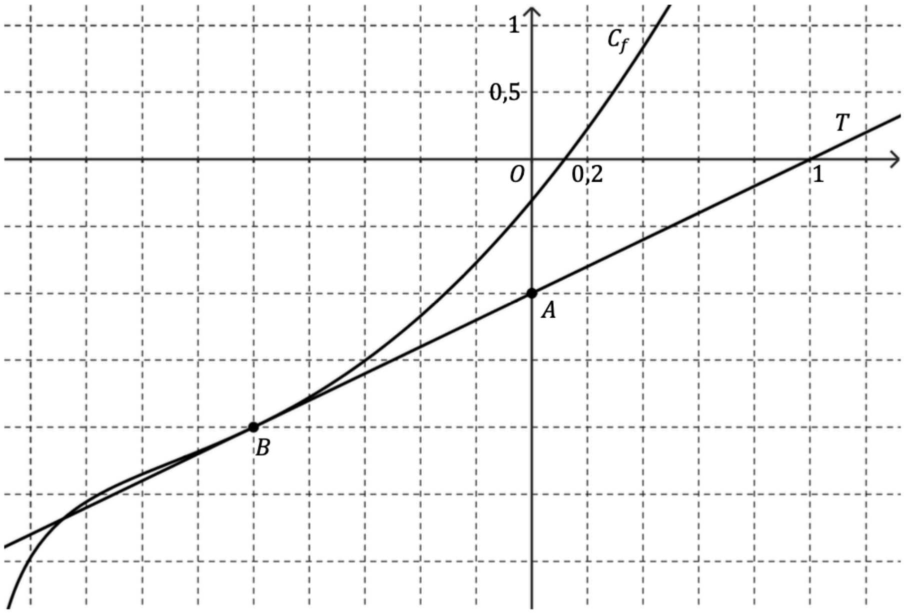
2.

a. Express in terms of q the $\lim$ of f in $+\infty$.

b. It is worth noting that the chlorine level observed on Wednesday, June 19th was equal to $0{,}7$ $mg.L^{-1}$ We hope that the chlorine level will stabilize in the long term around $2\ mg.L^{-1}$ Determine the values of C and q so that these two conditions are met.

Exercise 3 (6 points)

We consider a function f defined and twice differentiable on$]-2\,;+\infty[$. On note C_f its representative curve in an orthogonal coordinate system of the plane,f' its derivative andf'' its second derivative. The curve is plotted below.C_f and its tangent T to the point B abscissa -1. It is specified that the right T passes through the point $A(0\,;-1)$



Part A: Study of the graph.

Using the graph, answer the questions below.

1. To specify $f(-1)$ and $f'(-1)$.

2. The curveC_f is it convex on its domain of definition? Justify your answer.

3. Conjecture the number of solutions to the equation $f(x) = 0$ and give a rounded value to 10^{-1} close to a solution.

Part B: Study of the function.

We consider that the function f is defined on$]-2\,;+\infty[$ about $f(x) = x^2 + 2x - 1 + \ln(x+2)$, or ln denotes the natural logarithm function.

1. Determine the limit of the function by calculation f in -2. Interpret this result graphically. We assume that $\lim_{x\to+\infty} f(x) = +\infty$.

2. Show that for all $x > -2$, $f'(x) = (2x^2 + 6x + 5)/(x+2)$

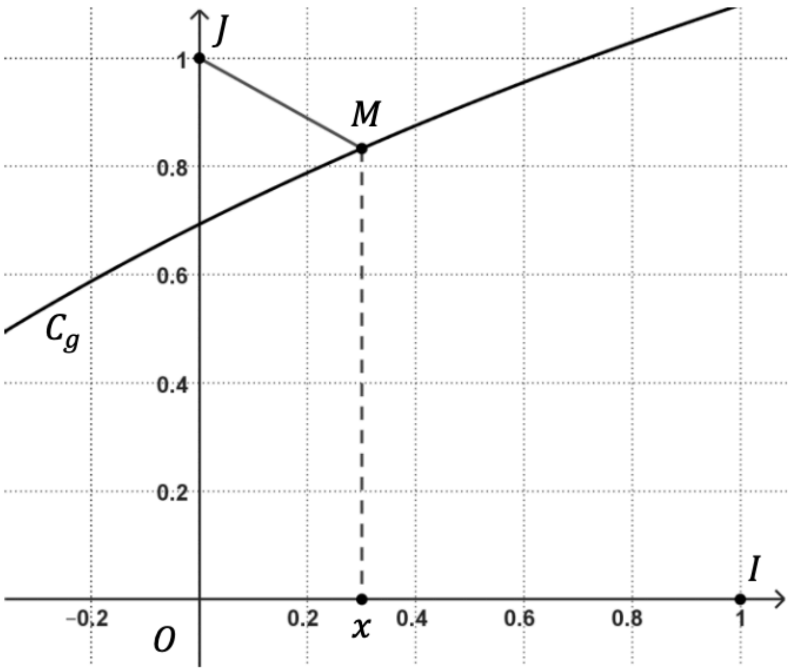
3. Study the variations of the function f on $]-2\,;+\infty[$ then draw up its complete table of variations.

4. Show that the equation $f(x) = 0$ admits a unique solution a on $]-2\,;+\infty[$ and give a rounded value of a has 10^{-2} close.

5. Deduce the sign of $f(x)$ on $]-2\,;+\infty[$.

6. Show that C_f admit a single inflection point and determine its x-coordinate.

Part C: a minimum distance.



Either g the function defined on $]-2\,;+\infty[$ about $g(x) = \ln(x + 2)$.

On note C_g its representative curve in an orthonormal coordinate system$(O;I,J)$, shown opposite.

Either M a point of C_g abscissa x..

The goal of this section is to determine for what value of x the distance JM is minimal.

We consider the function h defined on $]-2\,;+\infty[$ about $h(x) = JM^2$.

1. Justify that for all $x > -2$, on a : $h(x) = x^2 + [\ln(x + 2) - 1]^2$.

2. We admit that the function h is differentiable on $]-2 ; +\infty[$ and we note h' its derivative function. We also accept that for all real $x > -2$, $h'(x) = 2\, f(x) \,/\, (x + 2)$

or f is the function studied in part B.

a. Draw up the table of variations of h on $]-2 ; +\infty[$.
Limits are not requested.

b. Deduce that the value of x for which the distance JM is minimal is a or a is the real number defined in question 4 of part B.

3. Note M_a the point of C_g abscissa a.

a. Show that $\ln(a + 2) = 1 - 2a - a^2$.

b. Deduce that the tangent to C_g to the point M_a and the right (JM_a) are perpendicular.
We can use the fact that, in an orthonormal coordinate system, two lines are perpendicular when the product of their slopes is equal to -1.

Turn the page

Exercise 4 (4 points)

For each of the following statements, indicate whether it is true or false. Each answer must be justified. An unjustified answer will not receive any points.

In space equipped with an orthonormal coordinate system, we consider

following points: $A(2; 0; 0)$ $B(0; 4; 3)$ $C(4; 4; 1)$, $D(0; 0; 4)$

and $E(-1; 1; 2)$

Affirmation 1 : the points A, C and D define a plane P equation $8x - 5y + 4z - 16 = 0$.

Affirmation 2 : the points$A$, B, C and D are coplanar.

Affirmation 3 : the straight lines(AC) And (BH) are intersecting.

It is accepted that the plane (ABC) has the Cartesian equation $x - y + 2z - 2 =$
0.

Affirmation 4 : the point H is the orthogonal projection of the point D on the plane (ABC).